

Quiz 9

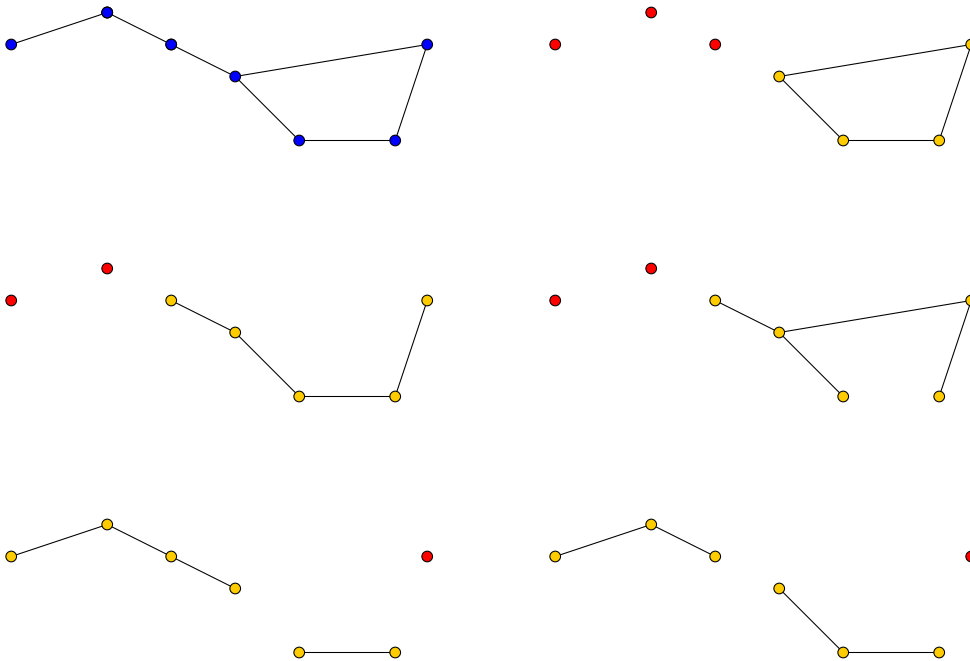
1. Enumerate the connected subgraphs of K_4 that have exactly 2 edges. (5 pts.)

Solution: There are 12, having edges

$$\begin{aligned} &\{ab, ac\} \quad \{ba, bc\} \quad \{ca, cb\} \quad \{da, db\} \\ &\{ab, ad\} \quad \{ba, bd\} \quad \{ca, cd\} \quad \{da, dc\} \\ &\{ac, ad\} \quad \{bc, bd\} \quad \{cb, cd\} \quad \{db, dc\} \end{aligned}$$

2. Find a representative for each of the isomorphism classes of subgraphs with exactly four edges in the "Ursa Major" graph below. (6 pts.)

Solution: Original graph shown below with blue vertices. For each of the other 5 graphs, include 0, 1, 2, or 3 of the **red** vertices for different isomorphism classes. Hence, total number coming from each graph is one more than the number of red dots for that graph. Total number of isomorphism classes among subgraphs of Ursa Major with exactly four edges is equal to $4 + 3 + 3 + 2 + 2 = 14$.



3. Write YES if the graph below has the specified property and NO otherwise: (3 pts. each)

$$\mathcal{G} = (\{a, b, c, d, e\}, \{ab, ac, bc, cd, ce\}).$$

- (a) has a closed path between two vertices, **Yes. $abca$ is a closed path**
- (b) has a vertex with degree one and a vertex with degree three, **No. The degree sequence is 4, 2, 2, 1, 1**
- (c) is bipartite. **No. It has an odd cycle**