

Worksheet #5

1. Let A and B be sets. If $|A| = 5 = |B|$, how many onto functions $f : A \rightarrow B$ are there? What if $|A| = 5$ and $|B| = 4$? Can you find a general formula for the case that $|A| = n$, $|B| = m$, where $n > m$? What about $n = m$? And for $n < m$?
2. Six friends walk into a bar...
 - (a) If each friend hugs each other friend, how much love (many hugs) occur(s)?
 - (b) How many different ways are there for the friends to sit at the bar (in a straight line, consecutively)?
 - (c) If the friends choose instead to sit at a round table, how many arrangements of seats are there (we consider 2 arrangements the same if each person has the same neighbor to the left)?
 - (d) How many arrangements are there if again they choose to sit at a round table, but this time we consider 2 arrangements the same if each person is sitting between the same 2 people?
 - (e) Do you see a relationship between (b), (c), and (d)? Explain, and conclude that the numbers in parts (b) and (c) will be even for any number of friends greater or equal to 3.
 - (f) Amongst the six friends, there is a couple and an inner-circle of 3 friends (neither member of the couple is a member of the inner-circle). Repeat parts (b),(c), and (d) with the added restriction that the couple sits tandem and the inner-circle sits consecutively.