Practice Exam 1

1. Prove that

$$1 + 2 + 2^2 + \cdots 2^n = 2^{n+1} - 1.$$

2. Prove or disprove:

$$2 + 4 + 6 + 8 + \cdots + 2n = (n-1)(n+2).$$

3. Valid? Prove or disprove.

$$(a)\quad \begin{array}{c} p \to q \\ \underline{q \vee r} \\ r \to (\neg q) \end{array} \qquad (b)\quad \begin{array}{c} p \to q \\ (\neg r) \vee (\neg q) \\ \underline{\quad r \quad} \\ (\neg p) \end{array}$$

4. Valid? Prove or disprove.

If I work hard, then I earn lots of money.
If I don't pay high taxes, then I don't work hard.
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If I work hard, then I pay high taxes.

5. True or False questions.

(i) If $p \wedge q$ is true, then $p \vee q$ is true.

(ii) If $p \to q$ is true and $q \to p$ is true, then p is logically equivalent to q.

(iii) If $\mathcal{A}$ is a tautology and $\mathcal{B}$ is a contradiction, then $\mathcal{A} \wedge (\neg \mathcal{B})$ is a tautology.

(iv) If $\mathcal{A} \iff \mathcal{B}$ and $\mathcal{C}$ is any statement, then $(\mathcal{A} \to \mathcal{C}) \iff (\mathcal{B} \to \mathcal{C})$.

(v) If the premises of an argument are all contradictions, then the argument is valid.

(vi) The statement $(p \to q) \leftrightarrow (q \wedge (r \to s)$ evaluates to TRUE when all the atomic statements p, q, r, s are true.

6. In the math department there are 30 personal computers (PCs).

20	have A drives,
8	have 19-inch monitors,
25	are running Windows XP,
20	have at least two of these properties,
6	have all three properties.

(a) How many PCs have at least one property?

(b) How many have none of these properties?

(c) How many have exactly one?

7. How many ways can you get a total of 6 when rolling two dice?

8. How many three digit numbers contain the digits 2 and 5 but not 0, 3, or 7?

9. In a group of 29 people, how many people must there be whose birthdays are in the same month?