Math 2602 | Finite and Linear Math | Fall '14

Quiz 11*

See second page.

1. Consider the graphs $\mathcal{G}$ and $\mathcal{H}$ defined as

$$\mathcal{G} = (\{a,b,c,d,e\}, \{ab, ac, ad, bc, ce, de\}),$$
$$\mathcal{H} = (\{v,w,x,y,z\}, \{vw, vx, wy, xy, xz, yz\}).$$

(a) Are $\mathcal{G}$ and $\mathcal{H}$ isomorphic? You need **not** justify your answer. (2 pts.)

YES.

(b) Is the map $\phi : \{a,b,c,d,e\} \to \{v,w,x,y,z\}$ defined below an isomorphism from $\mathcal{G}$ to $\mathcal{H}$? Justify your answer for full credit. (5 pts.)

$\boxed{\text{NO}}$

$$\phi(a) = y, \quad \phi(b) = z, \quad \phi(c) = x,$$
$$\phi(d) = v, \quad \phi(e) = w.$$

The map ϕ is bijective but NOT edge preserving since

$ad \in E(\mathcal{G})$ but $\phi(ad) = \phi(a)\phi(d) = yv \notin E(\mathcal{H})$.

So ϕ does NOT induce a map from $E(\mathcal{G})$ to $E(\mathcal{H})$.

(c) Is the map $\psi : \{a,b,c,d,e\} \to \{v,w,x,y,z\}$ defined below an isomorphism from $\mathcal{G}$ to $\mathcal{H}$? Justify your answer for full credit. (5 pts.)

$$\psi(a) = x, \quad \psi(b) = v, \quad \psi(c) = y,$$
$$\psi(d) = v, \quad \psi(e) = w.$$

$\boxed{\text{No.}}$ $\Psi(d) = \Psi(b) = v$ so Ψ is NOT one-to-one.

Also, z is not in the image of Ψ so Ψ is NOT onto either.

Hence, Ψ is NOT bijective.

2. The models for two unspecified graphs are shown below. Prove that the graphs whose models are shown are isomorphic or prove that no isomorphism exists. (8 pts.)

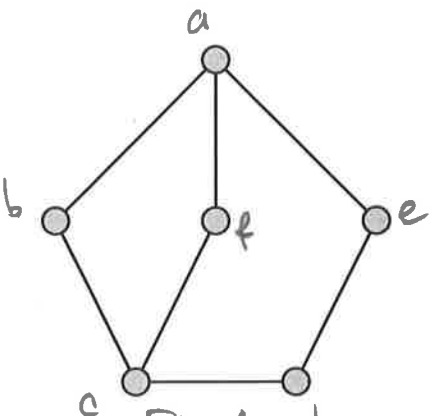

Fig 1.

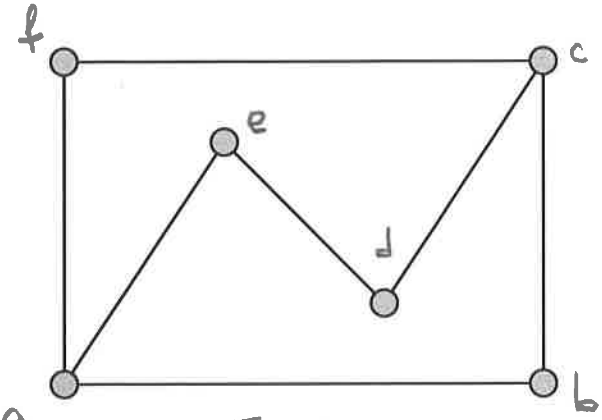

Fig 2.

Proof.

Let G_1 denote the graph whose unlabeled model is shown in Fig 1.

Let G_2 denote the graph whose unlabeled model is shown in Fig 2.

Then $G_1 \simeq \tilde{G}$ and $G_2 \simeq \tilde{G}$ $(\star)$

where $\tilde{G} = (\{a,b,c,d,e,f\}, \{ab, ae, af, bc, cd, cf, de\})$.

$\star$: (this is clear since we have labeled the models consistent with the definition of $\tilde{G}$).

So, $G_1 \simeq \tilde{G}$ and $\tilde{G} \simeq G_2$ by symmetry of isomorphism,

hence $G_1 \simeq G_2$ by transitivity of isomorphism.

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