

Quiz 2

The quizzes are designed to be 15 minutes. If necessary, you can take an extra 2-3 minutes just let your TA know if that will be necessary. Answer all the questions and be sure to justify your answer. Solutions that have no justification may receive deductions in points. Incorrect answers with some good justification may receive partial credit. Please attempt to be clear and concise in your response.

1. Prove that if both a and b are rational, then $2a - b$ is rational. (10 pts.)

let $n, m, h, k \in \mathbb{Z}$

a is rational $\therefore a = \frac{n}{m}$

$$2a - b = 2\left(\frac{n}{m}\right) - \frac{h}{k}$$

b is rational $\therefore b = \frac{h}{k}$

$$2a - b = \frac{2nk - hm}{mk}$$

both $2nk - hm$ & $mk \in \mathbb{Z} \therefore 2a - b$ is rational \square

2. Find the contrapositive of the statement: The product ab is irrational because either a or b is irrational. Is the statement true? (5 pts.)

If either a or b is irrational then product ab is irrational

Contrapositive: If product ab is rational then a and b is rational.

3. Is the following statement true or false? Justify your answer. (5 pts.)

There exists a prime number p such that for every prime number q , $q < p$.

false, \exists infinitely many prime

$\therefore \exists q > p$