

Quiz 4

1. Is the following argument valid? If valid, give a proof. If invalid, provide a counter-example. (8 pts.)

$$(1) \frac{(p \wedge q) \leftrightarrow q}{q \rightarrow p}$$

Valid. If q is true, then by the assumption (1)

$p \wedge q$ is true, so p is true.

Hence, if q is true then p is true.

If q is false, then the conclusion is true.

In both cases the argument is valid.

2. Prove that for every integer $n \geq 1$ the following equality holds. (12 pts.)

$$\sum_{i=1}^n \frac{1}{i(i+1)} = \frac{n}{n+1}$$

Proof. By induction

basis step: when $n=1$ we have

$$\sum_{i=1}^1 \frac{1}{i(i+1)} = \frac{1}{1(1+1)} = \frac{1}{2} \quad \text{which equals} \quad \frac{1}{1+1} = \frac{1}{2} \quad \checkmark$$

induction step: Assume $\sum_{i=1}^k \frac{1}{i(i+1)} = \frac{k}{k+1}$.

$$\begin{aligned} \text{Then} \quad \sum_{i=1}^{k+1} \frac{1}{i(i+1)} &= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)} = \frac{k(k+2) + (k+1)}{(k+1)(k+2)} \\ &= \frac{k^2 + 2k + k + 1}{(k+1)(k+2)} = \frac{k^2 + 3k + 1}{(k+1)(k+2)} \\ &= \frac{(k+1)^2}{(k+1)(k+2)} = \frac{k+1}{k+2}, \quad \text{as desired} \end{aligned}$$