

Midterm 1 Review Worksheet

1. If it does not walk like a duck and it does not talk like a duck, then it is not a duck.

(a) Find the atomic statements from the compound statement above.

(b) Find the negation, contrapositive, and converse of the compound statements.

(c) Construct a truth table for the compound statement.

2. Prove the following argument is valid:

$$\frac{p \wedge q}{r \rightarrow (\neg p)} \quad \neg r$$

3. In front of Alice and Bob are $n \geq 1$ stones sitting in a pile. Alice and Bob play the following game: In a sequence of alternating turns, each player picks up either one or two stones from the pile. The last player to pick up stones wins the game. Assume Alice goes first. Prove that if n is *not* a multiple of 3, then Alice can win the game (i.e. Alice has a winning strategy). Prove that if n is a multiple of 3, then Bob can win the game.

4. Let $n \geq 2$, and let x_1, x_2, \dots, x_n be real numbers in the interval $[0, 1)$. Prove that there exists indices $i \neq j$ such that

$$|x_i - x_j| < \frac{1}{n-1}.$$

5. A class of $2n$ students are forced to work on an assignment in pairs. How many ways can the students organize themselves into pairs?