Practice Exam 1

1. *Exploring: probability, distribution, random variable, expectation.* Consider the space which consists of the first 6 letters of the alphabet $S = \{a, b, c, d, e, f\}$. What does it mean to say that P is a probability on S?

(i) Let P_1 be the probability giving the uniform distribution on S. Describe P_1. What is $P_1(a)$? What is $P_1(\text{vowels})$?

(ii) Let P_2 be the probability for which each and any vowel has twice the probability of each and any consonant. (*Hint:* $P_2(a) = 2P_2(b)$, *for example*) What is $P_2(S)$? $P(b)$? What is $P_2(\text{vowels})$? What is $P_2(\{a, f\})$? $P_2(\{a, b, c\})$?

(iii) Let X be the random variable defined for $x \in S$ by

$$X(x) = \begin{cases} 0 & \text{if } x \text{ is a vowel,} \\ 1 & \text{if } x \text{ is a consonant.} \end{cases}$$

Find $E(X)$ in the two cases, using probability P_1 and probability P_2 separately. Repeat for the random variable $Y(x) = i$ if x is the i-th letter of the alphabet.

Recall: $E(X) = \sum_{x \in S} xP(x)$ *in this situation. Indeed, if* f *is a probability mass function and* u *is a payment function, then* $E(u(X)) = \sum_{x \in S} u(x)f(x)$*, and in this problem we have set* $u(x) := X(x)$ *and* $f(x) := P(X = x)$.

2. *Exploring: Probability mass function, expectation, mean, variance.* Let the random variable X have the probability mass function

$$f(x) = \frac{(|x|+1)^2}{9}, \qquad x = -1, 0, 1.$$

(i) Recall that $f(x) = P(X = x)$. What is $P(X = 1)$? What is $P(X \geq 0)$?

(ii) Compute $E(X), E(X^2)$, and $E(3X^2 - 2X + 4)$. Compute the variance σ^2 of the random variable X. Recall $\sigma^2 = E\left((X-\mu)^2\right)$ where $\mu = E(X)$.

(iii) Repeat the problem for f defined by $f(-1) = \frac{1}{3}$, $f(0) = f(-1) = \frac{1}{6}$.

3. *Exploring: counting problems, conditional probabilities, independence, mutually exclusive events.* Un urn contains 10 balls: three of which are blue, and seven of which are green. Let an experiment consist of picking two balls, one at a time, from the urn without replacement. Describe the sample space of the experiment, and then find the probability of each possible outcome. Finally, let E be the event "the first ball chosen is blue" and F be the event "the second ball chosen is green".

(i) Find $P(E)$, $P(F)$, and $P(E \cap F)$. Define *mutually exclusive.* Are E and F mutually exclusive?

(ii) Find $P(E|F)$. Are E and F independent events?