Math 3215 **Exam 2** Summer 2014

Instructor: Sal Barone

Name: ______________________________

GT username: ________________________

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Circle or box your answers.
5. Good luck!

Page	Max. Possible	Points
1	40	
2	30	
3	30	
Total	100	

1. The lengths of caterpillars in my backyard are normally distributed with average length 10 cm and standard deviation 2 cm. What is the probability that a randomly selected caterpillar has length less than 7 cm? If I collect 8 caterpillars, find the probability that exactly 3 of them has length less than 7 cm, and you may assume that the length of each caterpillar is independent of the others. (15 pts.)

Solution: Let X be the length of a caterpillar. Then $X \sim N(10, 4)$ and we construct $Z = \frac{X-10}{2}$ and look up $P(X < 7) = P(Z < \frac{7-10}{2}) = P(Z < -1.5) = 1 - P(Z < 1.5)$, which from Table V is $1 - .9332 \approx \boxed{.07}$. Now the number of caterpillars under 7 cm has binomial distribution, hence 3 of them are under 7 cm exactly

$$\binom{8}{3}(.07)^3(.93)^5 \approx \boxed{.01336}.$$

□

2. In the World Cup a goal is scored on average, for all games, once every 20 minutes. We can think of the scoring of a goal from the start of a game over time as an approximate Poisson process. Find the probability that the first goal of a game is scored in the first 10 minutes. Find the probability that exactly two goals occur in the first 60 minutes. Let Y be the waiting time for the second goal. How is Y distributed? Find $E(Y)$. (25 pts.)

Solution: Let X be the number of goals in a time interval of length 10 minutes (our *unit interval*). Then, X has the Poisson distribution with $\lambda = .5$, and we have

$$P(X \geq 1) = 1 - P(X < 1) = 1 - f(0) = 1 - \frac{.5^0 e^{-.5}}{0!} \approx \boxed{.3935}.$$

We can also instead think of W as the *waiting time* for the first goal. Then W has an exponential distribution with $\lambda = 1$ with unit interval 20 minutes. We compute $P(W < .5)$ since .5 of a unit interval is 10 minutes. We get $P(W < .5) = \int_0^{.5} e^{-x}\, dx = -e^{-x}\big|_0^{.5} = -(e^{-.5} - 1) = .3935$, so our answers agree :)

Let Z be the number of goals in a time interval of length 60 minutes (our *unit interval* for this part of the problem). Then, Z has the Poisson distribution with $\lambda = 3$, and we have

$$f(2) = \frac{3^2 e^{-3}}{2!} \approx \boxed{.224}.$$

Let Y be the waiting time for the fourth goal. Then Y has the gamma distribution with $\lambda = 1/20$, $\theta = 20$ where our unit interval is one minute and $\alpha = 2$. We have $\mu = \alpha\theta = 2 * 20 = \boxed{40}$.

□

3. Let X be a continuous random variable which is uniformly distributed on the interval $[-5, 5]$. Find $P(|X| < 2)$. Find c such that $P(X < c) = .2$. (15 pts.)

Solution: We have $P(|X| < 2) = P(-2 < X < 2) = P(X < 2) - P(X \leq -2) = \frac{2-(-5)}{5-(-5)} - \frac{-2-(-5)}{5-(-5)} = \frac{7-3}{10} = \boxed{.4}$. Also, we solve $P(X < c) = .2$ for c to obtain

$$P(X < c) = \frac{c - (-5)}{5 - (-5)} = .2 \quad \Longrightarrow \quad c + 5 = 10 * .2 \quad \Longrightarrow \quad \boxed{c = -3}.$$

□

4. Let X be the number of errors in each 100 foot section of a very long computer tape. Assume that X has a Poisson distribution with mean 2.5. Let W equal the number of feet before the first error is found. How is W distributed? Find the mean and variance of W. How many flaws would you expect to see in 15 feet of tape? (15 pts.)

Solution: The continuous random variable W has an exponential distribution with mean $\mu = \theta = 100 * 1/2.5 = \boxed{40}$ and variance $\sigma^2 = \theta^2 = \boxed{1600}$. We expect to see $15/100 * 2.5 = \boxed{.375 \text{ errors}}$ in 15 feet of tape.

□

5. Suppose the moment generating function of a continuous random variable X is

$$M(t) = e^{4t+6t^2}.$$

How is X distributed? Find μ and σ^2 of X. (15 pts.)

Solution: The random variable X is normally distributed with mean $\boxed{\mu = 4}$ and variance $\boxed{\sigma^2 = 12}$.

□

6. A biased coin is twice as likely to show heads than tails after being flipped. Consider a sequence of Bernoulli trials where you flip the coin until you see a tails. Find the probability that you get a tails on the first flip. Find the probability that you first see a tails on the third flip. If you keep flipping the coin after seeing the first tails, find the probability that you see exactly 4 tails in 10 flips. (15 pts.)

Solution: The coin is biased, with heads twice as likely as tails, so if q is the probability of heads and $p = 1 - q$ is the probability of tails we have $q = 2(1-q)$ and solving for q we get $3q = 2$ or $q = 2/3$ and $p = 1/3$. Thus the probability you get a tails on the first flip is $\boxed{p = 1/3}$. The number X of the first tails follows a geometric distribution with probability of success (getting a tails) $p = 1/3$, so the chance you get tails on the third flip is

$$f(3) = (1/3)(2/3)^2 = 4/27 \approx \boxed{.15}.$$

The number of tails you see in 10 flips follows a binomial distribution, so the chance you see exactly 4 tails is

$$\binom{10}{4}(1/3)^4(2/3)^6 \approx \boxed{.228}.$$

□