

Instructor: Sal Barone

Name: _____

GT username: _____

1. No books or notes are allowed.
2. You may use ONLY NON-GRAPHING and NON-PROGRAMABLE scientific calculators. All other electronic devices are not allowed.
3. Show all work and fully justify your answer to receive full credit.
4. Circle or box your answers.
5. Good luck!

Page	Max. Possible	Points
1	40	
2	40	
3	20	
Total	100	

1. Let X be the birth weight of babies born in Atlanta. Assume that $E(X) = 7.2$ and $\text{Var}(X) = 3.2^2$. If a random sample of $n = 16$ birth weights are recorded, approximate the probability that the mean weight of the sample is greater than 8. (20 pts.)

2. The joint p.m.f. of X and Y is $f(x, y)$, $x, y \in \{1, 2\}$, defined by $f(1, 1) = 5/8$, $f(1, 2) = (2, 1) = f(2, 2) = 1/8$. Find the marginal p.m.f.'s of X and Y . Also, compute the covariance $\text{Cov}(X, Y)$ of X and Y . Is it possible to determine whether or not X and Y are independent based on your computation of the covariance? If so, are they independent? (20 pts.)

3. Suppose X_1, X_2, X_3 are normally distributed random variables with the given mean and variances: $X_1 \sim N(3, 1)$, $X_2 \sim N(7, 4)$, and $X_3 \sim N(0, 1)$. Describe as accurately as possible the distributions of the two random variables $Y = X_1 + X_2 + X_3$ and \bar{X} . (20 pts.)

4. The p.d.f. of X is $f(x) = \theta x^{\theta-1}$, $0 < x < 1$, where $\theta > 0$ is a parameter. Let $Y = -\theta \ln X$. How is Y distributed? *Hint: The function $y = -\theta \ln x$ is decreasing in x , so the following slight modification of the change of variables formula will work $g(y) = f(v(y)) \cdot (-v'(y))$ where $v(y)$ is the inverse function of $y = u(x) = -\theta \ln x$.* (20 pts.)

5. The time X in minutes of a visit to a doctor by a patient is modeled by a gamma p.d.f. with $\alpha = 1.5$ and $\theta = 10$. The patients wait in the waiting room until the doctor is free, as he sees several patients in the same day. Suppose that you are such a patient and have two patients ahead of you in the waiting room when you arrive. Assuming independence of the visit lengths of the patients ahead of you, set up an integral that calculates the probability that you must wait longer than 60 minutes. You do NOT need to evaluate the integral you set up, but you must simplify it as much as possible. (20 pts.)