Practice Exam 3

1. Consider the function $f(x, y) = \alpha(x + 2y)$ if $0 < x, y < 1$ and $f(x, y) = 0$ otherwise. Find a value α such that f is a joint p.d.f. Is this choice unique? Find the marginal p.d.f.'s of X and Y. Find the covariance of X and Y. Is it possible to determine whether or not X and Y are independent **only** using what you just found out about the covariance?

2. State the Central Limit Theorem. A soda company has installed a new machine to fill its bottles. The machine was poorly installed, so the amount the machine dispenses has a standard deviation of 0.1 liters, and the mean is 2 liters. Approximate the probability that after 40 bottles are filled that the the sample mean is at least 1.9 liters.

3. The cumulative distribution function of a continuous random variable is given by $F(x) = 1 - e^{-3x}$, $x > 0$. Find the p.d.f. of this random variable.

4. Cars arrive at a toll booth at a rate of 4 calls every 6 minutes. Assume these cars arrive as a Poisson process. What is the probability that the 5th car arrives at exactly 6 minutes and 45 seconds?

5. Let X and Y be random variables on the space $S = \{(0,0),(1,1),(1,-1),(2,0)\}$ with joint p.m.f. $f(x,y) = 1/4$. Compute the covariance and correlation coefficient. Are X and Y independent? Can you tell if they are independent **only** from the correlation coefficient?

6. Let $X_1, \ldots, X_8$ be a random sample from a distribution having p.m.f. $f(x) = (x+1)/6$, $x = 0, 1, 2$. What is the p.m.f. of $Y_1 = X_1 + X_2$? What is the p.m.f. of $Y_2 = X_3 + X_4$? What about $Y = X_1 + X_2 + X_3 + X_4$ and $W = X_1 + \ldots + X_8$?