Homework 1b: Due 5/29/14

1. You roll two dice. Let X denote the sum of the dice. Compute the probability mass function $f(x)$ and plot a probability histogram.

Solution: The space of X is $S = \{2, 3, \ldots, 12\}$. The p.m.f. of X is $f(x) = \frac{x-1}{36}$ for $x = 2, \ldots, 7$ and $f(x) = \frac{14-x}{36}$ for $x = 8, \ldots, 12$, and note that $f(x) = f(14 - x)$ for $x = 2, \ldots, 7$. The histogram is roughly "normal".

□

2. Let μ and σ^2 denote the mean and variance of a random variable X. Determine $E[(X - \mu)/\sigma]$ and $E[((X - \mu)/\sigma)^2]$.

Solution: From the linearity of expectation we have $E[(X - \mu)/\sigma] = \frac{1}{\sigma}(E(X) - \mu) = 0$. Also, $E[((X - \mu)/\sigma)^2] = \frac{1}{\sigma^2}E((X - \mu)^2) = 1$.

□

3. A hat is filled with 6 chips. Three are blue, two are red, and one is yellow. Determine the random variable X such that $X(blue) = 0$, $X(red) = 1$, and $X(yellow) = 2$. Assume each chip is equally likely to be drawn. Compute the mean, variance, and standard deviation of this probability distribution.

Solution: The sample space is $S = \{b_1, b_2, b_3, r_1, r_2, y\}$, and the random variable X is described by $X(b_i) = 0$ for $i = 1, 2, 3$, $X(r_j) = 1$ for $j = 1, 2$, and $X(y) = 2$. Since the distribution is uniform we have $P(x) = 1/6$ for each $x \in S$. Computing the mean we have $E(X) = 3 * 0 * 1/6 + 2 * 1 * 1/6 + 1 * 2 * 1/6 = 4/6 = 2/3$, and the variance is

$$E((X - \mu)^2) = 3 * (2/3)^2 * 1/6 + 2 * (1/3)^2 * 1/6 + (4/3)^2 * (1/6) = (1/54)(12 + 2 + 16) = 5/9.$$

So the standard deviation is $\sqrt{5}/3$.

□

4. In a recent poll 65% of Americans disapprove of how their government works. Suppose this is true in general about all Americans and let X be the number of people who disapprove in a random sample of size 15. How is X distributed? Find $P(X \geq 10)$, $P(X \leq 10)$ and $P(X = 10)$. Find the mean, variance and standard deviation of X.

Solution: The probability of getting 10 people out of 15 to disagree is $\binom{15}{10}(.65)^{10}(.35)^5$. Similarly, the probability of getting n people out of 15 to disagree is $\binom{15}{n}(.65)^n(.35)^{15-n}$. Thus $P(X = 10) = .212$, and

$$P(X \geq 10) = \sum_{n=10}^{15} \binom{15}{n}(.65)^n(.35)^{15-n} = .564,$$

and so $P(X \leq 10) = 1 - (P(X \geq 10) - P(X = 10)) = 1 - (.564 - .212) = 1 - .776 = .224$.

$\square$