Homework 2: Due 6/5/14

1. Let X be a random variable with binomial distribution $b(n,p)$, which means that the space of X is $S = \{0, 1, 2, \ldots, n\}$ and the p.m.f. $f(x)$ is $f(x) = \binom{n}{x} p^x (1-p)^{n-x}$. Show that $E(X) = np$ and $E\left((X-\mu)^2\right) = np(1-p)$. You may use without proof that $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$ (the *Binomial Theorem*), and I would suggest you show that $E\left((X-\mu)^2\right) = E(X(X-1)) + E(X) - \mu^2$ and examine the expression $E(X(X-1))$.

2. On a 8 question multiple choice exam there are five possible answers (a), (b), (c), (d), and (e) for each question, and exactly one answer is correct. What is the probability mass function of X the number of correct answers if a student answers randomly? What is the probability that the student who answers randomly gets the first and last question right, and every other question wrong? What is the probability that they get exactly 3 questions correct?

3. Using X from the Problem 2 what is the moment generating function of X? Show that the m.g.f. $M(t)$ satisfies $M'(0) = E(X)$ and $M''(0) = E(X^2)$. Find μ, σ, and σ^2 for this random variable using the formula from Problem 1.

4. Let X be the number of randomly selected people you must ask in order to find someone that has the same birthday as you. Assume each day is equally likely and ignore leap years. What is the p.m.f. of X? What is μ, σ^2 and σ of X? What is the probability that you have to ask more than 400 people? What about fewer than 300?

5. Consider X from the problem above except that X is the number of randomly selected people you must ask in order to find n people that have the same birthday as you. Find the *p.m.f.* $f(x)$ of X and the m.g.f. $M(t)$ of X.