Homework 3b: Due 6/19/14

1. Define what it means for X to be a continuous random variable. How is the probability density function $f(x)$ used to calculate $P(X \leq x)$? What does the distribution function $F(x)$ measure?

2. Cars arrive randomly at a 200 second stoplight (*i.e.*, a stoplight which is red for 200 seconds). Let X be the amount of time a randomly selected car has to wait at the light before it turns green. If X is $U(0, 200)$, meaning that it is uniformly distributed on the interval $[0, 200]$, find the p.d.f. of X, and find the probability that the car must wait longer than 2 minutes. What is the probability that the car has to wait between 1 and 2 minutes at the light?

3. Suppose the lifespan of a certain type of electrical component follows an exponential distribution with a mean life of 50 days. If X denotes the life of this component (in days) then find $P(X > x)$, which is a function of x the number of days before failure. Find $P(X > 20)$ and also find the conditional probability $P(X > 40|X > 20)$, the probability that the component lasts 40 days given that it lasts 20 days. Are these probabilities equal? Is an exponential a good model for the lifespan of a component?

4. If 10 observations are taken independently from a chi-square distribution with 19 degrees of freedom, find the probability that exactly 2 of the 10 sample items exceed 30.14.

5. Cars arrive at a toll booth at a mean rate of three cars every 4 minutes according to a Poisson process. What is the probability that there are fewer than two cars in a 4 minute period? Find the probability that the toll booth collector has to wait longer than 10 minutes to collect the 9th toll.