Homework 4: Due 6/26/14

1. Let the joint p.m.f. of X and Y be defined by

$$f(x,y) = \frac{x+2y}{33}, \quad x = 1,2,\ y = 1,2,3.$$

Find the marginal p.d.f. of X and that of Y. Find $P(X > Y)$, $P(Y > X)$ and $P(X+Y=2)$. Are X and Y independent?

Solution: The marginal p.d.f.'s of X and Y are

$$f_1(x) = \sum_{y=1,2,3} f(x,y) = \frac{3x+12}{33},$$
$$f_2(x) = \sum_{x=1,2} f(x,y) = \frac{3+4y}{33}.$$

To find $P(X > Y)$ we note that $x > y$ exactly when $x = 2$ and $y = 1$, so $P(X > Y) = f(2,1) = \boxed{\frac{5}{33}}$. Now $P(Y > X) = \sum_{y>x} f(x,y) = f(1,2) + f(1,3) + f(2,3) = \frac{5+7+8}{33} = \boxed{\frac{20}{33}}$. Also, $P(X+Y=2) = \sum_{x+y=2} f(x,y) = f(1,1) = \boxed{\frac{3}{33}}$. Clearly, $f(x,y) \neq f_1(x)f_2(y)$, so X and Y are independent.

□

2. With the joint p.d.f. defined above in Problem 1, find the means μ_X, μ_Y, the variances σ_X^2, σ_Y^2 and the correlation coefficient ρ.

Solution: We compute

$$\mu_X = \sum_{x=1,2} x f_1(x) = (1)\frac{15}{33} + (2)\frac{18}{33} = \boxed{\frac{51}{33}},$$
$$\mu_Y = \sum_{y=1,2,3} y f_2(y) = (1)\frac{7}{33} + (2)\frac{11}{33} + (3)\frac{15}{33} = \boxed{\frac{74}{33}}.$$

3. Select an even integer uniformly at random from the list $\{0, 2, 4, 6, 8\}$ and then select an integer uniformly at random from the list $\{0, 1, 2, 3, 4\}$. Let X be the number selected from the first list and Y equal the sum of the two numbers. Find the joint p.m.f. of X and Y and the marginal p.d.f. of each. Are X and Y independent?

4. Let X and Y be continuous random variables with joint p.d.f.

$$f(x, y) = 2, \quad 0 \leq y \leq x \leq 1.$$

Sketch the domain of f. Find the marginal p.d.f.'s of X and Y and compute $\mu_X, \mu_Y, \sigma_X^2, \sigma_Y^2, \mathrm{Cov}(X, Y)$, and ρ.