## Homework 5: Due 7/3/14

**1.** Let $X$ and $Y$ be continuous random variables with joint/marginal p.d.f.'s

$$\begin{aligned} f(x,y) &= 2, && 0 \le x \le y \le 1, \\ f_1(x) &= 2(1-x), && 0 \le x \le 1, \\ f_2(y) &= 2y, && 0 \le y \le 1. \end{aligned}$$

Find the conditional p.d.f. $h(y|x)$ and the conditional probability $P\left(\frac{1}{2} \le Y \le \frac{3}{4} \;\middle|\; X = \frac{1}{4}\right)$. What is the expected value of $Y$ when $X = \frac{1}{4}$?

**2.** Let $X$ and $Y$ be discrete random variables with joint p.m.f.

$$f(x,y) = \frac{x+y}{32}, \qquad x = 1,2, \quad y = 1,2,3,4.$$

Find the marginal p.m.f.'s of $X$ and $Y$ and the conditional p.m.f.'s $g(x|y)$ and $h(y|x)$. Find $P(1 \le Y \le 3 \mid X = 1)$ and $P(Y \le 2 \mid X = 2)$. Finally, find $E(Y \mid X = 1)$ and find $Var(Y \mid X = 1)$.

**3.** Let $W$ equal the weight of a box of oranges which is supposed to weight 1-kg. Suppose that $P(W < 1) = .05$ and $P(W > 1.05) = .1$. Call a box of oranges light, good, or heavy depending on if $W < 1$, $1 \leq W \leq 1.05$, or $W > 1.05$, respectively. In $n = 50$ independent observations of these boxes, let $X$ equal the number of light boxes and $Y$ the number of good boxes.

Find the joint p.m.f. of $X$ and $Y$. How is $Y$ distributed? Name the distribution and state the values of the parameters associated to this distribution. Given $X = 3$, how is $Y$ distributed? Determine $E(Y \mid X = 3)$ and find the correlation coefficient $\rho$ of $X$ and $Y$.

**4.** Let $X$ have the uniform distribution $U(0, 2)$ and let the conditional distribution of $Y$, given that $X = x$, be $U(0, x)$. Find the joint p.d.f. $f(x, y)$ of $X$ and $Y$, and be sure to state the domain of $f(x, y)$. Find $E(Y|x)$.

**5.** The *support* of a random variable $X$ is the set of $x$-values such that $f(x) \neq 0$. Given that $X$ has p.d.f. $f(x) = x^2/3$, $-1 < x < 2$, what is the support of $X^2$? Find the p.m.f. of the random variable $Y = X^2$.

**6.** Let $X_1, X_2$ denote two independent random variables each with the $\chi^2(2)$ distribution. Find the joint p.d.f. of $Y_1 = X_1$ and $Y_2 = X_1 + X_2$. What is the support of $Y_1, Y_2$ (i.e., what is the domain of the joint p.d.f., where $f(y_1, y_2) \neq 0$)? Are $Y_1$ and $Y_2$ independent?