Homework 5b: Due 7/10/14

1. Let X_1, X_2, X_3 be a random sample from a distribution with p.d.f. $f(x) = 2e^{-2x}$, $0 < x < \infty$. This means that X_1, X_2, X_3 are independent random variables each with the same p.d.f. $f(x)$, or said even another way that they are independent identically distributed random variables (written i.i.d.). Find the probability $P(0 < X_1 < 2, 2 < X_2 < 4, 4 < X_3 < 6)$. What is the joint p.d.f. of X_1, X_2, X_3? What is the probability that exactly one of X_1, X_2, X_3 is in the range $0 < x < 2$ and exactly one is in the range $2 < x < 4$ and exactly one is in the range $4 < x < 6$?

2. Let X_1, X_2 be independent random variables with respective binomial distributions $b(3, .25)$ and $b(4, .5)$. Find $P(X_1 = 2, X_2 = 3)$ and $P(X_1 + X_2 = 5)$.

3. Let X_1, X_2, X_3 be i.i.d. random variables with Poisson distributions with mean $\lambda = 3$. Find the moment generating function of $Y = X_1 + X_2 + X_3$. How is Y distributed?

4. Let $\bar{X}$ denote the mean of a random sample of size 25 from a distribution whose p.d.f. is $f(x) = x^3/4$, $0 < x < 2$. It is easy to show that $\mu = 8/5$ and $\sigma^2 = 8/75$. Use the central limit theorem to approximate $P(1.4 < \bar{X} < 1.7)$.

5. Let X_i, $1 \leq i \leq n$ be a random sample of size n from the continuous uniform distribution $U(0,1)$ with p.d.f. $f(x) = 1$. Find the mean μ_i and variance σ_i^2 of X_i, $1 \leq i \leq n$. Find the mean and variance of $\bar{X} = \frac{X_1 + \cdots + X_n}{n}$. Approximate $P(\bar{X} \leq n/2)$ using the central limit theorem.