Homework 6: Due 7/24/14

1. Let $f(x;\theta) = \frac{1}{\theta}\, x^{(1-\theta)/\theta}$, $0 < x < 1$ and $0 < \theta < \infty$. Find the maximum likelihood estimator $\hat{\theta}$ of θ. Is $\hat{\theta}$ unbiased?

2. Let $X_1, \ldots, X_n$ be a random sample from $b(1,p)$, so that $Y = \sum_{i=1}^{n} X_i$ is $b(n,p)$. Show that $\bar{X} = Y/n$ is an unbiased estimator of p. Show that $\mathrm{Var}(\bar{X}) = p(1-p)/n$ and that $E(\bar{X}(1-\bar{X})/n) = (n-1)\frac{p(1-p)}{n^2}$. Is $\bar{X}(1-\bar{X})/n$ an unbiased estimator for $\mathrm{Var}(\bar{X})$? Find c such that $c\bar{X}(1-\bar{X})$ is an unbiased estimator for $\mathrm{Var}(\bar{X})$.

3. A sample of $n = 7$ of the lengths of caterpillars is given below. Assume that these observations are from a normal distribution $N(\mu, \sigma^2)$. Find the sample mean and sample variance of these observations. Do the sample mean and sample variance give good approximations to μ and σ^2 *(try to use the concept of estimators in your answer)*? Find an approximate 95% confidence interval for μ and an approximate 80% confidence interval for μ. Explain in words what each interval is doing. Which interval has the greater length? Why?

17.5	14.5	15.2	14.0	17.3	18.0	13.8