

Quiz 1

Answer the questions. Be sure to justify your answer do not simply write the answer down. Use complete sentences where appropriate and scrap paper if needed (on the desk at the front of class, scrap paper must be turned in but will not be graded for work). Circle or box your answer where appropriate. You may ask questions about the wording of a question or to clarify the instructions.

1. Define "relative frequency" and "uniform distribution". Use complete sentences.

Solution:

- *Relative frequency:* The relative frequency of an outcome in a random experiment is the number of times the outcome occurs divided by the total number of trials of the experiment. The relative frequency of an event is the number of times the event occurs divided by the number of trials the experiment was performed.
- *Uniform distribution:* Recall that a probability P is a function which assigns a real number $P(A)$ to each event A in the sample space S , and such that this function satisfies the three conditions of being a probability. A probability has a uniform distribution if each outcome is equally likely, in particular $P(\{a\})$ is a constant for every $a \in S$.

2. A pizza parlor offers three sizes of pizza with 8 possible toppings, one of which is pepperoni. How many different large, three topping pizzas where one of the toppings is pepperoni could the pizza parlor make? If three friends each order a random pizza (meaning that the size, toppings are randomly chosen and are equally likely) what is the chance that exactly two of them get the same size pizzas?

Solution: There are $\binom{7}{2}$ different large three topping pizzas with pepperoni since you have to choose 2 of the remaining 7 toppings to add as the other two toppings.

$$\text{And } \boxed{\binom{7}{2} = 21}.$$

If three friends order a random pizza and exactly two of them get the same size then among the three sizes, two of three sizes are chosen and one of those two sizes is repeated. There are $\binom{3}{2} \cdot \binom{2}{1} = 3 \cdot 2 = 6$ ways to accomplish this and there are $3!/2 = 3$ ways to permute who gets which sizes, and there are $3^3 = 27$ ways to make three ternary choices, so the probability is

$$\boxed{18/27 \approx .67}.$$

3. Three friends Abe, Braum, and Cia go to subway and order their favorite sandwich (which are all different). However, the worker making the subs forgets who ordered which sub and hands them back randomly. What is the probability that they all receive the sub they ordered? What is the probability that nobody gets the right sub?

Solution: There is exactly one way for the three of them to all receive the right sub, and there is $3 \cdot 2 \cdot 1 = 3! = 6$ ways to permute 3 objects (in this case the objects being permuted are the 3 subs if the guys are standing in order, or they are the 3 guys if the subs are being handed out one at a time, etc.). So the probability is $\boxed{1/6 \approx 17\%}$.

There are $3! = 6$ ways for the three people to get the three subs. Of these, nobody gets the right sub in two situations, depicted below

Abe \rightarrow Braum \rightarrow Cia \rightarrow Abe,
Abe \rightarrow Cia \rightarrow Braum \rightarrow Abe.

(if this isn't obvious just list out the 6 possibilities it's not that hard). You can read an arrow Abe \rightarrow Braum as "Abe's sandwich gets handed to Braum". So the probability is $\boxed{2/6 \approx 33\%}$.