

Quiz 2

Answer the questions. Be sure to justify your answer do not simply write the answer down. Use complete sentences where appropriate and scrap paper if needed (on the desk at the front of class, scrap paper must be turned in but will not be graded for work). Circle or box your answer where appropriate. You may ask questions about the wording of a question or to clarify the instructions.

1. Let the moment generating function of X be

$$M(t) = \frac{1}{6}e^t + \frac{1}{3}e^{2t} + \frac{1}{2}e^{3t}.$$

Find μ , σ^2 , and σ . What is the p.m.f. of X ? (10 pts.)

Solution: We know $\mu = M'(0)$ and $\sigma^2 = M''(0) - (M'(0))^2$, where the moment generating function is given above. We compute,

$$\begin{aligned}M'(t) &= \frac{1}{6}e^t + \frac{2}{3}e^{2t} + \frac{3}{2}e^{3t}, \\M''(t) &= \frac{1}{6}e^t + \frac{4}{3}e^{2t} + \frac{9}{2}e^{3t}.\end{aligned}$$

So, $\mu = 14/6 = 7/3$ and $\sigma^2 = 6 - (7/3)^2 = 5/9$, and $\sigma = \sqrt{5}/3$. Finally,

$$f(x) = \begin{cases} 1/6 & \text{if } x = 1, \\ 1/3 & \text{if } x = 2, \\ 1/2 & \text{if } x = 3. \end{cases}$$

□

2. A certain company that manufactures electric fans knows that it takes 70 Watts of electricity for the fan to run correctly. Assume that 8% of the fans the company manufactures are defective. If you inspect fans one after one until you find a defective one, what is the probability that the 10th fan inspected is the first defective fan you inspect. Find an expression for the probability that one of the first 10 fans you inspect is defective, but do not evaluate it.

Make sure to appropriately define a random variable X and probability mass function $f(x)$.
(10 pts.)

Solution: Let X be the number of the first defective fan you inspect. Then X follows a geometric distribution, with $p = 8\%$ and $q = 92\%$. So,

$$f(x) = (.08)(.92)^{x-1},$$

and $f(10) = (.08)(.92)^{10-1} = .03777$. The expression

$$P(X \leq 10) = \sum_{x=1}^{10} f(x) = \sum_{x=1}^{10} (.08)(.92)^{x-1}$$

expresses the quantity that is the probability that one of the first 10 fans you inspect is the first defective fan you inspect.

□