

Quiz 3

Answer the questions. Be sure to justify your answer do not simply write the answer down. Use complete sentences where appropriate and scrap paper if needed (on the desk at the front of class, scrap paper must be turned in but will not be graded for work). Circle or box your answer where appropriate. You may ask questions about the wording of a question or to clarify the instructions.

1. The mean of a Poisson random variable X is $\mu = 4$. Compute

$$P(\mu - \sigma < X < \mu + \sigma).$$

(10 pts.)

Solution: The p.m.f. $f(x)$ of X is $f(x) = \frac{4^x e^{-4}}{x!}$ with $\lambda = 4$. Since $\lambda = \mu = \sigma^2 = 4$, we have $\sigma = 2$. So,

$$\begin{aligned} P(\mu - \sigma < X < \mu + \sigma) &= P(4 - 2 < X < 4 + 2) \\ &= P(2 < X < 6) \\ &= P(X < 6) - P(X \leq 2) \\ &= P(X \leq 5) - P(X \leq 2) \\ &= .785 - .238 \\ &= \boxed{.547}. \end{aligned}$$

□

2. A magician is practicing a trick where he throws a knife to pop a balloon, but the balloon magically does not pop the first two times he hits it. He needs to be able to hit the balloon three times for the trick to work. If the probability that he can hit the balloon is 80%, then what is the probability that he can hit the balloon successfully 3 times in 5 attempts? Let X be the number of trials needed to observe the third success after independent Bernoulli trials each of probability .8 of successfully hitting the target. How is X distributed? (10 pts.)

What I meant to ask would have been better stated as “what is the probability that it takes exactly 5 attempts to make 3 successful hits”?

Solution: The discrete random variable X follows a negative binomial distribution

$$f(x) = \binom{x-1}{3-1} p^3 q^{x-3}, \quad p = .8, \quad q = 1 - p = .2.$$

We are given that $f(5) = .80$ from which we see that

$$\binom{4}{2} (.8)^3 (1 - .8)^2 = 6 (.8)^3 (.2)^2 = \boxed{.1229}.$$

□