Quiz 4

Answer the questions. Be sure to justify your answer do not simply write the answer down. Use complete sentences where appropriate and scrap paper if needed (on the desk at the front of class, scrap paper must be turned in but will not be graded for work). Circle or box your answer where appropriate. You may ask questions about the wording of a question or to clarify the instructions.

1. Suppose X and Y are independent discrete random variables with marginal p.m.f.'s $f_1(x)$ and $f_2(y)$, respectively. If $f_1(3) = .4$ and $f_2(6) = .3$, then what is $f(3, 6)$? (5 pts.)

Solution: Since $f(x, y) = f_1(x)f_2(y)$ when X and Y are independent, we have $f(3, 6) = f_1(3)f_2(6) = (.4)(.3) = \boxed{.12}$.

□

2. Let the joint p.m.f. of X and Y be $f(x, y) = 1/4$ for $(x, y) \in S = \{(0, 0), (1, 1), (1, -1), (2, 0)\}$. Calculate $\text{Cov}(X, Y)$ and the correlation coefficient ρ. Are X and Y independent? (10 pts.)

Solution: To find $\text{Cov}(X, Y) = E((X - \mu_X)(Y - \mu_Y)) = E(XY) - \mu_X\mu_Y$ we first need to find μ_X and μ_Y. For that, it is useful to know the marginal p.m.f.'s of X and Y which are

$$f_1(0) = 1/4, \quad f_1(1) = 1/2, \quad f_1(2) = 1/4,$$
$$f_2(0) = 1/2, \quad f_2(1) = 1/4, \quad f_2(-1) = 1/4.$$

So,

$$\mu_X = E(X) = 0(1/4) + 1(1/2) + 2(1/4) = 1,$$
$$\mu_Y = E(Y) = 0(1/2) + 1(1/4) - 1(1/4) = 0.$$

Now,

$$E(XY) = 0 \cdot 0(1/4) + 1 \cdot 1(1/4) + 1 \cdot -1(1/4) + 2 \cdot 0(1/4) = 0.$$

So, $\text{Cov}(X, Y) = 0 - 1 \cdot 0 = \boxed{0}$, and hence $\rho = \text{Cov}(X, Y)/\sigma_X\sigma_Y = \boxed{0}$, too (no matter what the standard deviations are! so we don't have to calculate them). However, X and Y are clearly $\boxed{\text{dependent}}$ since, for example, if $x = 2$ then that forces $y = 0$, or, alternately, since $f(x, y) \neq f_1(x)f_2(y)$ for all pairs x, y.

□

3. Let X and Y be continuous random variables having the joint p.d.f.

$$f(x,y) = x + y, \quad 0 < x < 1,\ 0 < y < 1.$$

Find the marginal p.d.f.'s of X and Y. (5 pts.)

Solution: We must integrate to find, for example, the marginal p.d.f. $f_1(x)$ of X.

$$f_1(x) = \int_0^1 f(x,y)\, dy = \int_0^1 x + y\, dy = x + \frac{y^2}{2}\bigg|_0^1 = \boxed{x + \frac{1}{2}}.$$

Since the domain and the joint p.d.f. are symmetric, we have $\boxed{f_2(y) = y + \frac{1}{2}}$, too.

$\square$