Quiz 5

1. A certain fast food chain is doing a promotion where whenever you buy a hamburger you get one of three stickers, and if you collect all three stickers you win free hamburgers for a year. Suppose the three stickers are called A, B, C and the probability of a certain sticker being A, B, or C is $p_1 = .12$, $p_2 = .85$, and $p_3 = .03$, respectively. If I buy 100 hamburgers and have 100 stickers, what is the probability that 10 of them are A and 5 of them are C stickers? (10 pts.)

Solution: The joint p.m.f. of X_1, X_2, X_3 the number of stickers of type A, B, C, respectively, is given by the joint p.m.f. of a trinomial distribution with $n = 100$,

$$f(x_1, x_2, x_3) = \frac{100!}{x_1!x_2!x_3!}(.12)^{x_1}(.85)^{x_2}(.03)^{x_3}, \qquad x_1 + x_2 + x_3 = 100.$$

So, the probability that $X_1 = 10$ and $X_3 = 5$ is given by

$$f(10, 85, 5) = \frac{100!}{10!85!5!}(.12)^{10}(.85)^{85}(.03)^5 = \boxed{.01146}.$$

□

2. Let X be a continuous random variable with p.d.f. $f(x) = |x|$, $-1 < x < 1$. What is the p.d.f. $g(y)$ of $Y = X^2$? Be sure to state the support of g. How is Y distributed? (10 pts.)

Solution: We first find the cumulative distribution function $G(y)$ of Y.

$$G(y) = P(Y \le y) = P(X^2 \le y) = P(-\sqrt{y} \le X \le \sqrt{y}) = \int_{-\sqrt{y}}^{\sqrt{y}} |x| \, dx.$$

To deal with this integral we need to split it up into two pieces, since $f(x) = |x|$ is defined piecewise. We have

$$\begin{aligned} G(y) &= \int_{-\sqrt{y}}^{\sqrt{y}} |x| \, dx \\ &= \int_{-\sqrt{y}}^{0} -(x) \, dx + \int_{0}^{\sqrt{y}} x \, dx \\ &= \int_{0}^{-\sqrt{y}} x \, dx + \int_{0}^{\sqrt{y}} x \, dx = \frac{(-\sqrt{y})^2}{2} + \frac{(\sqrt{y})^2}{2} = y. \end{aligned}$$

So, $G(y) = y$, and hence $g(y) = G'(y) = 1$, for $0 < y < 1$. Hence, $Y \sim \mathrm{U}(0, 1)$ is uniformly distributed on the interval $(0, 1)$.

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