MATLAB #1 - Exploration 3

Please see demo.txt file on my website for basic examples. Proper use of comments and sections is required for full credit (remember: "%" comments out a line and "%%" followed by a space creates a section - see the sample pdf on my website for details).

1. A simple calculation: What is $11 + 31$? To find this, simply enter the command "`11 + 31`". Running this section will return the correct answer in the command line.

2. A linear system with no free variables: Consider the linear system

$$x_1 + 2x_2 + 3x_3 = 115$$
$$2x_1 - x_2 + 3x_3 = 1421$$
$$4x_1 - x_2 + 12x_3 = 4214$$

   You'll solve this linear system in a few ways:

   (i) First, enter the coefficient matrix for this system as the variable "A", and the right-hand side as the column vector "b". Next, enter the command "`A \ b`", which outputs the (in this case, unique) solution to this linear system.

   (ii) Second, enter the augmented matrix as the variable "M". In the next line, run the command `rref(M)`, which outputs the row reduced echelon form of M. If you did everything right, you should find that the solution to the system is the same as that you found in part (i).

3. Another linear system:

   (i) Consider the linear system

$$4x_1 - 7x_2 = -33$$
$$-3x_1 + 8x_2 = 44$$
$$-3x_1 + 7x_2 = 37$$

   Note that there are three equations in only two unknowns. Write the solution values of x_1, x_2 in the comments. Why is the solution NOT the augmented column of the RREF in this case? *(Hint: make sure that your x vector satisfies $Ax = b$.)*

   (ii) Explain in the comments using precise terminology from the class the answer to the following questions: (1) What are the conditions for the **augmented column of the RREF** to be the solution to the system. (2) In the case of a unique solution to a system with *more equations than variables*, how should the solutions be obtained

from the RREF? (3) In the case of a consistent system where there are *more variables than equations*, what information is contained in the augmented column of the RREF?

4. A linear system with a free variable: Using only what we have so far, MATLAB does not provide parametric solutions to an underdetermined system of equations. To see this, consider the linear system consisting of the first two equations in the linear system in the previous Problem 2.

$$x_1 + 2x_2 + 3x_3 = 115$$
$$2x_1 - x_2 + 3x_3 = 1421$$

Enter the backslash command `A \ b` (for appropriate A, b) to obtain a particular solution. On the other hand, we know from class that, if such a system with more variables then there are equations is a consistent system, then there must be infinitely many solutions: provide an explanation as to why this is true.

At best, using the tools at our disposal, we will have to read the set of solutions off of the RREF. For this, enter the appropriate augmented matrix and take its RREF using MATLAB. Based on this, identify vectors `v`, `w` (that is, declare them in your Matlab script) so that a general solution is of the form $v + aw$, where a is any real number.

For your Gradescope submission we need a .pdf file from the MATLAB $\boxed{\text{PUBLISH}}$ button (either publish straight to .pdf - or publish first to .html then put it in a browser and print to .pdf from the browser), but only after you have completed and verified the below:

- All your code compiles correctly using the $\boxed{\text{RUN}}$ command, and

- all required comments (using %) and headers (using %%) are present.

Please read the MATLAB intro file carefully at
`https://people.math.gatech.edu/~sbarone7/MATLAB_intro.pdf` including the portion at the end about submitting your MATLAB work to Gradescope. See the sample pdf file
`https://people.math.gatech.edu/~sbarone7/test_demo_submission.pdf` for an example of the kind of file we want to see in Gradescope.

NOTE: **Any submission other than a .pdf as described above will be penalized.**