

## Practice Exam 1

1. A certain professor has a data bank of 15 easy and 20 hard problems for a ten problem true/false exam. How many ways can the professor select problems 1 through 10 so that there are 5 easy problems and 5 hard problems.

Choose 5 easy  $\binom{15}{5}$

Choose 5 hard  $\binom{20}{5}$

$$\boxed{\binom{15}{5} + \binom{20}{5} \neq 10!}$$

arrange the 10 problems  $10!$

2. A student who is totally unprepared for the exam that the professor made above decides to guess randomly on the 10 true/false questions without even reading them. How many different ways can the student answer the questions?

ten binary choices T/F

$$\boxed{2^{10}}$$

3. Determine the coefficient of  $x^5y^8$  in the binomial expansion of  $(x+y)^{13}$ .

$\boxed{\binom{13}{5}}$  or  $\binom{13}{8}$  ok too (they are equal)

ok answer?

$\boxed{1287}$  preferred answer



4. The Greek alphabet contains 24 letters. How many three letter words are there in this alphabet in which no letter appears twice? How many three letter words in total?



$$\binom{24}{3} * 3! = 24 * 23 * 22 =$$

$$\boxed{12144} \text{ best answer}$$

5. A class of 15 science majors and 60 humanities majors is doing a project. In how many ways can the professor select a group of 12 students consisting of 3 science majors and 9 humanities majors to head the project?

$$\boxed{\binom{15}{3} \binom{60}{9}} = 455 * 1.47 \times 10^{10} ?? \text{ big \#}$$

preferred answer.

6. Let  $U = \{a, b, c, \dots, z\}$  be the set of 26 letters of the English alphabet. How many subsets of  $U$  contain all the vowels and any number of consonants (including zero consonants).

5 vowels  $\{a, e, i, o, u\}$

21 consonants  $\{b, c, d, f, g, h, j, k, \dots, z\}$

subset  
must  
contain

$\{a, e, i, o, u\}$

and may contain any of

$\{b, c, \dots, z\}$

So 21 binary choices. Ans.

$$\boxed{2^{21}}$$



7. How many words have exactly six different letters but no vowels.

21 consonants.

$$\boxed{\binom{21}{6} * 6!} = 37090080$$

8. How many 5-card poker hands contain exactly 3 aces?

Choose the aces  $\binom{4}{3} = 4$

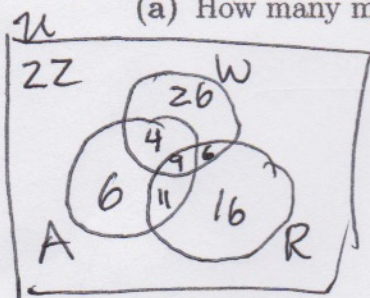
Choose other two cards  $\frac{48 * 47}{2}$

total 4512

$$\boxed{4 * \frac{48 * 47}{2}}$$

9. The 100 members of Earth Club were surveyed to find out whether what the club's priorities should be: clean water, clean air, or recycling. The responses were 45 for clean water, 30 for clean air, 42 for recycling, 13 for both clean air and clean water, 20 for clean air and recycling, 15 for clean water and recycling, and 9 for all three.

- (a) How many members thought the priority should be clean air only?



$$\boxed{6} = |A \cap W \cap R'|$$

- (b) How many members thought the priority should be clean water or clean air, but not both?

$$|(A \cup W) \cap (A \cap W)'| = 6 + 11 + 26 + 6 = \boxed{49}$$

- (c) How many members thought the priority should be recycling but not clean air?

$$|R \cap A'| = \boxed{22}$$

- (d) How many members thought the priority should be none of these three issues?

$$|(A \cup W \cup R)'| = \boxed{22}$$