

## Quiz 3 (11 am)

1. Find the solutions of the matrix equation  $Ax = 0$  where  $A$  is the matrix below. For full credit you **must** write your answer in parametric vector form. (10 pts.)

$$A = \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ -1 & 0 & 2 & -1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} \nearrow x & \searrow y & \searrow z & \searrow u & \searrow v \\ \begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$\text{So } \begin{cases} x = -3y \\ y = s \text{ (free)} \\ z = -y \\ u = r \text{ (free)} \\ v = 0 \end{cases}$$

} parametric  
equation  
form of solns.

$$X = \begin{bmatrix} x \\ y \\ z \\ u \\ v \end{bmatrix} = r \begin{bmatrix} -3 \\ 0 \\ -1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} : & : & : & : \\ : & : & : & : \\ : & : & : & : \end{bmatrix}$$

2. True or false. Assume the matrix  $A$  has 3 rows and 4 columns, so it's size is  $3 \times 4$ , meaning the corresponding system has 3 equations and 4 unknowns. (2 pts. each)

- (a)  TRUE /  FALSE If  $A$  has three pivot positions, then the equation  $Ax = 0$  has a non-trivial solution.
- (b)  TRUE /  FALSE If  $A$  has three pivot positions, then the equation  $Ax = 0$  has the trivial solution.
- (c)  TRUE /  FALSE If  $A$  has three pivot positions, then the equation  $Ax = b$  is always consistent for all  $b \in \mathbb{R}^3$ .
- (d)  TRUE /  FALSE If  $x$  is a nontrivial solution to  $Ax = 0$ , then every entry in  $x$  is nonzero.
- (e)  TRUE /  FALSE The homogeneous system  $Ax = 0$  has infinitely many solutions.