Quiz 4 (11 am)

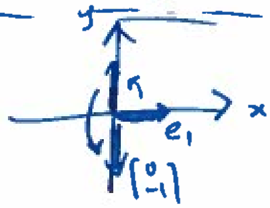
1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which associates to each $\mathbf{x} \in \mathbb{R}^2$ the vector obtained from $\mathbf{x}$ by first rotating $\mathbf{x}$ by 90° counter-clockwise and then reflecting the result about the horizontal x-axis. Find the standard matrix A of T as well as the image $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$.

Hint: the first column of A is $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right)$ and the second column of A is $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right)$. (4 pts. ea.)

$A = [T(e_1)\ T(e_2)]$

ANS. $A = \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}0 & -1\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

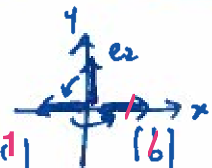
$e_1 = \begin{bmatrix}1\\0\end{bmatrix}$



$T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}0\\-1\end{bmatrix}$

$e_2 = \begin{bmatrix}0\\1\end{bmatrix}$

$T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\0\end{bmatrix}$



2. Determine whether the given vectors are linearly independent or linearly dependent. If the vectors are linearly dependent find a non-trivial linear combination of the vectors which give the zero vector. (8 pts.)

$$\underset{v_1}{\begin{bmatrix}1\\-1\\0\end{bmatrix}},\ \underset{v_2}{\begin{bmatrix}2\\2\\1\end{bmatrix}},\ \underset{v_3}{\begin{bmatrix}6\\2\\2\end{bmatrix}}$$

Solve $Ax=0$
if unique soln $x=0$
then lin independent.

ANS
$\boxed{\text{Linearly dependent!}}$
& $2v_1 + 2v_2 = v_3$

$$A = \begin{bmatrix}1 & 2 & 6\\-1 & 2 & 2\\0 & 1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 6\\0 & 4 & 8\\0 & 1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 6\\0 & 1 & 2\\0 & 1 & 2\end{bmatrix} \sim \begin{bmatrix}1 & 2 & 6\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}$$

2 pivots and 3 cols. $\Rightarrow$ free variable $\Rightarrow$ NOT l.i. ind

3. True or False section. (1 pt. each)

$A = \begin{bmatrix}\vdots & \vdots & \vdots\end{bmatrix}$

T / F If A is a 4×3 matrix with 3 pivots, then the columns of A are linearly independent.

T / F If $Ax = 0$ has the trivial solution, then the columns of A are linearly independent.

False T / F If the columns of A are linearly independent, then $Ax = b$ has a unique solution. could be inconsistent

T / F The linear transformation with standard matrix $\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}$ rotates vectors in $\mathbb{R}^2$ by 90° ~~counter-clockwise~~. Clockwise

$$\begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\-1\end{bmatrix}$$