Math 1552, Integral Calculus

Section 8.5: Partial Fractions

Evaluate the following integrals using any method we have learned.

1. $\int \frac{x+3}{(x-1)(x^2-4x+4)}dx$
2. $\int \frac{x+4}{x^3+x}dx$
3. $\int x^5 \ln(x)dx$
4. $\int \tan(x) \ln[\cos(x)]dx$
5. $\int \frac{x+2}{x+1}dx$
6. $\int \sqrt{25-x^2}dx$
7. $\int \tan^3(x) \sec^4(x)dx$
8. $\int x \tan^{-1}(x)dx$
9. $\int \frac{dx}{x\sqrt{1+x^2}}$
10. $\int \frac{x+1}{x^2(x-1)}dx$
11. $\int \frac{x+1}{x^2-4x+8}dx$

Answers

1. $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$ (partial fractions)

2. $4\ln|x| - 2\ln(x^2+1) + \tan^{-1}(x) + C$ (partial fractions)

3. $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$ (by parts)

4. $-\frac{1}{2}(\ln[\cos(x)])^2 + C$ (u-substitution)

5. $x + \ln|x+1| + C$ (long division)

6. $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right) + \frac{x\sqrt{25-x^2}}{2} + C$ (trig sub)

7. $\frac{1}{4}\tan^4(x) + \frac{1}{6}\tan^6(x) + C$ (trig identities)

8. $\frac{x^2}{2}\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + C$ (by parts)

9. $-\ln\left|\frac{\sqrt{1+x^2}}{x} + \frac{1}{x}\right| + C$ (trig sub)

10. $-2\ln|x| + \frac{1}{x} + 2\ln|x-1| + C$ (partial fractions)

11. $\frac{1}{2}\ln|x^2-4x+8| + \frac{3}{2}\tan^{-1}\left(\frac{x-2}{2}\right) + C$ (rational function)