Math 1552, Integral Calculus

Section 10.1: Sequences

1. For each sequence below, find the l.u.b. and g.l.b., and determine if the sequence is monotonic.

$$\{\sin(n\pi)\}, \quad \left\{(-1)^{n+1}\frac{1}{5^n}\right\}, \quad \{\frac{n+1}{n}\}$$

For the first sequence, note that:

$$\{\sin(n\pi)\} = \sin(\pi), \sin(2\pi), \sin(3\pi), ... = 0, 0, 0, 0, ...$$

The sequence is constant and thus monotonic. Here, l.u.b.=g.l.b.=0.

For the second sequence,

$$\left\{(-1)^{n+1}\frac{1}{5^n}\right\} = \frac{1}{5}, -\frac{1}{25}, \frac{1}{125}, ...$$

Since the sequence alternates between positive and negative values, it is **not** monotonic. Here, l.u.b.=$\frac{1}{5}$ and g.l.b.=$-\frac{1}{25}$.

In the third sequence, we note that $a_n = 1 + \frac{1}{n}$, so:

$$a_{n+1} = 1 + \frac{1}{n+1} < 1 + \frac{1}{n} = a_n,$$

meaning the terms are decreasing. Thus, l.u.b.=2 (first term) and g.l.b.=1 (limit of the terms).

2. Determine whether or not each sequence converges. If so, find the limit.

(a) $\left\{\frac{2n^2}{\sqrt{9n^4+1}}\right\}$

$$\begin{aligned}\lim_{n\to\infty} \frac{2n^2}{\sqrt{9n^4+1}} &= \lim_{n\to\infty} \sqrt{\frac{4n^4}{9n^4+1}} \\ &= \sqrt{\lim_{n\to\infty} \frac{4n^4}{9n^4+1}} \\ &= \sqrt{\frac{4}{9}} = \frac{2}{3},\end{aligned}$$

so the sequence converges to $\frac{2}{3}$.

(b) $\left\{\left(1 - \frac{1}{8n}\right)^n\right\}$

$$\lim_{n\to\infty}\left(1-\frac{1}{8n}\right)^n = \lim_{n\to\infty}\left(1+\frac{-1/8}{n}\right)^n = e^{-1/8},$$

so the sequence converges to $e^{-1/8}$.

(c) $\left\{\frac{n!}{e^n}\right\}$

Recall that for $x > 0$: $\lim_{n\to\infty}\frac{x^n}{n!} = 0$, so $\lim_{n\to\infty}\frac{n!}{e^n} = \infty$, and the sequence **diverges**.

(d) $\left\{\left(\frac{n}{n+5}\right)^n\right\}$

Note that:

$$\begin{aligned}
\lim_{n\to\infty}\left(\frac{n}{n+5}\right)^n &= \lim_{n\to\infty}\frac{1}{\left(\frac{n+5}{n}\right)^n} \\
&= \lim_{n\to\infty}\frac{1}{\left(1+\frac{5}{n}\right)^n} \\
&= \frac{1}{\lim_{n\to\infty}\left(1+\frac{5}{n}\right)^n} \\
&= \frac{1}{e^5},
\end{aligned}$$

so the sequence **converges**.