

Math 1552: Derivative Review Relay

Find the derivative of each function below. DO NOT SIMPLIFY YOUR ANSWER.

1. $f(x) = \sin(\tan^{-1}(\sqrt{x^3 + 5x - 2}))$

$$f'(x) = \cos(\tan^{-1}(\sqrt{x^3 + 5x - 2})) \cdot \frac{1}{1+(x^3+5x-2)} \cdot \frac{3x^2+5}{2\sqrt{x^3+5x-2}}$$

2. $g(x) = \frac{3x^{1/4}e^{1/x}}{(x^4 - \frac{1}{3x})^5(3x^2+2)^4}$

$$\ln[g(x)] = \ln 3 + \frac{1}{4} \ln x + \frac{1}{x} - 5 \ln(x^4 - \frac{1}{3x}) - 4 \ln(3x^2 + 2), \text{ so: } \frac{g'(x)}{g(x)} = \frac{1}{4x} - \frac{1}{x^2} - \frac{5(4x^3 + \frac{1}{3x^2})}{x^4 - \frac{1}{3x}} - \frac{4(6x)}{3x^2+2}.$$

Then: $g'(x) = \frac{3x^{1/4}e^{1/x}}{(x^4 - \frac{1}{3x})^5(3x^2+2)^4} \left[\frac{1}{4x} - \frac{1}{x^2} - \frac{5(4x^3 + \frac{1}{3x^2})}{x^4 - \frac{1}{3x}} - \frac{24x}{3x^2+2} \right].$

3. $h(x) = (\ln x)^x$

$$\ln[h(x)] = x \ln(\ln x), \text{ so } \frac{h'(x)}{h(x)} = \ln(\ln x) + x \cdot \frac{1}{\ln x} \cdot \frac{1}{x}. \text{ Then: } h'(x) = (\ln x)^x \left[\ln(\ln x) + \frac{1}{\ln x} \right].$$

4. $k(x) = \log_2(\log_5(\log_6(8^{-3x})))$

$$k'(x) = \frac{1}{\ln 2(\log_5(\log_6(8^{-3x})))} \cdot \frac{1}{\ln 5((\log_6(8^{-3x}))} \cdot \frac{1}{\ln 6(8^{-3x})} \cdot (-3)(\ln 8)(8^{-3x})$$

5. $s(t) = t^2 \csc^3(5t) \sec^5(8t)$

$$s'(t) = 2t \csc^3(5t) \sec^5(8t) - 15t^2 \csc^3(5t) \cot(5t) \sec^5(8t) + 40t^2 \csc^3(5t) \sec^5(8t) \tan(8t)$$