**Review for Test 3**

Math 1552, Integral Calculus

Sections 8.8, 10.1-10.5

1. Terminology review: complete the following statements.

(a) A geometric series has the general form

____________. The series converges when ____________ and diverges when ____________.

(b) A p-series has the general form ____________. The series converges when ____________ and diverges when ____________. To show these results, we can use the ____________ test.

(c) The harmonic series ____________ and telescoping series ____________.

(d) If you want to show a series converges, compare it to a ____________ series that also converges. If you want to show a series diverges, compare it to a ____________ series that also diverges.

(e) If the direct comparison test does not have the correct inequality, you can instead use the ____________ ____________ test. In this test, if the limit is a ____________ number (not equal to ____________), then both series converge or both series diverge.

(f) In the ratio and root tests, the series will ____________ if the limit is less than 1 and ____________ if the limit is greater than 1. If the limit equals 1, then the test is ____________.

(g) If $\lim_{n\to\infty} a_n = 0$, then what do we know about the series $\sum_k a_k$? ____________

(h) An integral is improper if either one or both limits of integration are ____________, or the function has a ____________ ____________ on the interval $[a, b]$.

(i) A **sequence** is an infinite ____________ of terms.

A sequence $\{a_n\}$ converges if: ____________.

(j) The smallest value that is greater than or equal to every term in a sequence is called the ____________. The largest value that is less than or equal to every term in the sequence is called the ____________. If both of these values are finite, then we say the sequence is ____________.

(k) A sequence is called monotonic if the terms are ____________, ____________, ____________, or ____________. If a sequence is both monotonic and bounded, then we know it must ____________.

2. Sum the series

$$\sum_{k=2}^{\infty} \frac{4^{2k} - 1}{17^{k-1}}.$$

3. Find the sum of the series

$$\sum_{k=1}^{\infty} \frac{1}{(2k-1)(2k+3)}.$$

4. Determine whether the following series converge or diverge. Justify your answers using the tests we discussed in class.

(a) $\sum_{k=1}^{\infty} \frac{e^k}{(1+4e^k)^{3.2}}$

(b) $\sum_{k=2}^{\infty} \left(\frac{k-5}{k}\right)^{k^2}$

(c) $\sum_{k=1}^{\infty} \frac{k^2 \cdot 2^{k+1}}{k!}$

(d) $\sum_{k=1}^{\infty} \frac{1}{1+2+3+\ldots+k}$

5. For each sequence, determine: (i) the l.u.b. and g.l.b.; (ii) whether the sequence is monotonic; (iii) whether the series converges or diverges, and the limit if it is convergent.

(a) $\left\{\left(\frac{n}{n+2}\right)^{3n}\right\}$

(b) $\left\{\frac{\cos(n\pi)}{4^n}\right\}$

(c) $\left\{(-1)^n \frac{n+2}{n+4}\right\}$

6. Determine if the improper intergral converges or diverges. If it converges, evaluate the integral.

(a)

$$\int_{2}^{\infty} \frac{x}{(x^2-1)^{3/2}}\,dx.$$

(b)

$$\int_{0}^{2} \frac{dx}{x^2-5x+6}.$$