Math 1552, Integral Calculus

Section 5.4: The Fundamental Theorem of Calculus

1. Evaluate the integrals:

(a) $\int_1^2 \frac{3x-5}{x^3}dx$.

$$\begin{aligned}
&= \int_1^2 \left(\frac{3}{x^2} - \frac{5}{x^3}\right) dx \\
&= \left[-\frac{3}{x} + \frac{5}{2x^2}\right]_1^2 \\
&= \left(-\frac{3}{2} + \frac{5}{8}\right) - \left(-\frac{3}{1} + \frac{5}{2}\right) \\
&= -\frac{3}{8}.
\end{aligned}$$

(b) $\int_1^3 |x-2|dx$.

$$\begin{aligned}
&= \int_1^2 (2-x)dx + \int_2^3 (x-2)dx \\
&= \left[2x - \frac{x^2}{2}\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^3 \\
&= \left[(4-2) - (2-\frac{1}{2})\right] + \left[(\frac{9}{2} - 6) - (2-4)\right] \\
&= 1.
\end{aligned}$$

2. Find $F'(2)$ for the function

$$F(x) = \int_{\frac{8}{x}}^{x^2} \left(\frac{t}{1-\sqrt{t}}\right) dt.$$

Using the 2nd FTC:

$$\begin{aligned}
F'(x) &= \frac{x^2}{1-\sqrt{x^2}} \cdot 2x - \frac{8/x}{1-\sqrt{8/x}} \cdot \left(-\frac{8}{x^2}\right) \\
&= \frac{2x^3}{1-|x|} + \frac{64}{x^3(1-\sqrt{8/x})}.
\end{aligned}$$

Then:

$$F'(2) = \frac{2 \cdot 2^3}{1-|2|} + \frac{64}{2^3 \cdot (1-\sqrt{8/2})} = -24.$$