Math 1552, Integral Calculus

Section 5.5-5.6: Integration by Substitution

1. Evaluate the integrals:

$$\int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$$

Solution: Let $u = \frac{1}{x}$, then $du = -\frac{1}{x^2} dx$ and the integral becomes:

$$-\int \sec u \tan u du = -\sec u + C = -\sec\left(\frac{1}{x}\right) + C.$$

$$\int \frac{\sin 3x - \cos 3x}{\sin 3x + \cos 3x} dx$$

Solution: Let $u = \sin 3x + \cos 3x$, then $du = (3\cos(3x) - 3\sin(3x))dx$ so $-\frac{1}{3} du = (\sin 3x - \cos 3x)dx$ and the integral becomes:

$$-\frac{1}{3} \int \frac{du}{u} = -\frac{1}{3} \ln|u| + C = -\frac{1}{3} \ln|\sin 3x + \cos 3x| + C.$$

$$\int \frac{1}{\ln(x^x)} dx$$

Solution: Note that $\ln(x^x) = x \ln x$ (properties of logs). Let $u = \ln x$, then $du = \frac{1}{x} dx$ and the integral becomes:

$$\int \frac{1}{u} du = \ln|u| + C = \ln|\ln x| + C.$$

2. Evaluate the following integrals:

$$\int \frac{e^{2x}}{\sqrt{4-3e^{2x}}} dx$$

Solution: Let $u = 4 - 3e^{2x}$, then $du = -6e^{2x} dx$ so $-\frac{1}{6} du = e^{2x} dx$. The integral becomes:

$$-\frac{1}{6} \int \frac{du}{\sqrt{u}} = -\frac{1}{3}\sqrt{u} + C = -\frac{1}{3}\sqrt{4-3e^{2x}}.$$

$$\int \frac{dx}{\sqrt{4-(x+3)^2}}$$

Solution: First, rewrite the integral. Pulling a 4 out of the denominator yields:

$$\frac{1}{2} \int \frac{dx}{\sqrt{1-\left(\frac{x+3}{2}\right)^2}}.$$

Now set $u = \frac{x+3}{2}$, then $du = \frac{1}{2} dx$ and the integral becomes:

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1}(u) + C = \sin^{-1}\left(\frac{x+3}{2}\right) + C.$$

$$\int \frac{5p^{\sqrt{x+1}}}{\sqrt{x+1}} dx, \quad p > 0$$

Solution: Let $u = \sqrt{x+1}$, then $du = \frac{1}{2\sqrt{x+1}} dx$ so $2du = \frac{1}{\sqrt{x+1}} dx$, and the integral becomes:

$$10 \int p^u du = \frac{10}{\ln p} p^u + C = \frac{10}{\ln p} p^{\sqrt{x+1}} + C.$$