Math 1552, Integral Calculus

Section 5.6: Area Between Curves

1. Suppose that y = f(x) and y = g(x) are both continuous functions on the interval [a, b]. Determine if each statement below is always true or sometimes false.

(a) Suppose that f(c) > g(c) for some number $c \in (a, b)$. Then the area bounded by f, g, x = a, and x = b can be found by evaluating the integral $\int_a^b (f(x) - g(x)) dx$.

(b) If $\int_a^b (f(x) - g(x)) dx$ evaluates to -5, then the area bounded by f, g, x = a, and x = b is 5.

(c) If f(x) > g(x) for every $x \in [a, b]$, then $\int_{a}^{b} |f(x) - g(x)| dx = \int_{a}^{b} (f(x) - g(x)) dx$.

2. Find the area bounded by the region between the curves $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$.

3. Find the area bounded by the region enclosed by the three curves $y = x^3$, y = -x, and y = -1.

4. Find the area bounded by the curves $y = \cos x$ and $y = \sin(2x)$ on the interval $\left[0, \frac{\pi}{2}\right]$.

5. Find the area of the triangle with vertices at the points (0,1), (3,4), and (4,2). USE CALCULUS.

Answers

1. (c) is true 2. $\frac{37}{12}$ 3. $\frac{5}{4}$ 4. $\frac{1}{2}$ 5. 4.5