Math 1553: Intro to Linear Algebra

Sections 2.8-2.9: Basis and Subspaces

Name:

1. Let $\mathbf{v}= \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\mathbf{w}= \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$. Find a vector $\mathbf{x}$ in $\Re^3$ so that the set $\{\mathbf{v}, \mathbf{w}, \mathbf{x}\}$ is a basis for $\Re^3$.

2. Find the coordinates of the vector $\mathbf{v}= \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ subject to the basis of $\Re^3$ given by $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} \right\}$.

3. Determine if the following statements are always true or sometimes false. If the statement is false, explain why.

(a) The empty set is a subspace of $\Re^n$.

(b) The set of all vectors $\mathbf{b}$ such that $A\mathbf{x}=\mathbf{b}$, where $\mathbf{x}$ is any vector in $\Re^n$ and A is an $m \times n$ matrix, is a subspace of $\Re^m$.

(c) For any set of vectors $\mathbf{v}_1,\ldots,\mathbf{v}_k \in R^n$, span$(\mathbf{v}_1,\mathbf{v}_2,\ldots,\mathbf{v}_k)$ is closed under scalar multiplication and addition.

4. For the matrix below, determine: (i) the rank and nullity; and (ii) a basis for the column space and nullspace.

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$