Math 1552, Integral Calculus

Sections 8.2, 8.3: Integration by Parts/Powers and Products of Trig Functions

Evaluate the following integrals using any of the integration techniques we have learned.

1. $\int \sin^5(2x)\cos^3(2x)dx$

Break off a cosine:

$$\begin{aligned}
\int \sin^5(2x)\cos^3(2x)dx &= \int \sin^5(2x)\cos^2(2x)\cos(2x)dx \\
&= \int \sin^5(2x)(1-\sin^2(2x))\cos(2x)dx \\
&= \int (\sin^5(2x)-\sin^7(2x))\cos(2x)dx \\
[u=\sin(2x)] &= \frac{1}{2}\int (u^5-u^7)du \\
&= \frac{1}{2}\left[\frac{u^6}{6}-\frac{u^8}{8}\right]+C \\
&= \frac{1}{12}\sin^6(2x)-\frac{1}{16}\sin^8(2x)+C.
\end{aligned}$$

2. $\int_1^e \frac{\sqrt{\ln x}}{x}dx$

u-substitution: Let $u=\ln x$, then $du=\frac{1}{x}dx$:

$$= \int_0^1 \sqrt{u}du = \frac{2}{3}u^{3/2}|_0^1 = \frac{2}{3}.$$

3. $\int (\ln x)^2 dx$

Integration by parts: let $u_1=(\ln x)^2$ and $dv_1=dx$. Then $du_1=\frac{2\ln x}{x}dx$ and $v_1=x$, so the integral becomes:

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \ln(x)dx.$$

By parts again: $u_2=\ln x$ and $dv_2=dx$, so $du_2=\frac{1}{x}dx$ and $v_2=x$:

$$\int (\ln x)^2 dx = x(\ln x)^2 - 2\left[x\ln x - \int 1dx\right] = x(\ln x)^2 - 2x\ln x + 2x + C.$$

4. $\int x^3 e^{x^2}\,dx$

Integration by parts: let $u = x^2$ and $dv = xe^{x^2}$. Then $du = 2xdx$ $v = \frac{1}{2}e^{x^2}$, so:

$$\begin{aligned}\int x^3 e^{x^2}\,dx &= \frac{x^2 e^{x^2}}{2} - \int xe^{x^2}\,dx \\ &= \frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2} + C.\end{aligned}$$

5. $\int \tan^4(x)dx$

$$\begin{aligned}\int \tan^4(x)dx &= \int \tan^2(x)\tan^2(x)dx \\ &= \int (\sec^2(x) - 1)\tan^2(x)dx \\ &= \int \tan^2(x)\sec^2(x)dx - \int \tan^2(x)dx \\ &= \int \tan^2(x)\sec^2(x)dx - \int (\sec^2(x) - 1)dx \\ &= \frac{1}{3}\tan^3(x) - \tan(x) + x + C.\end{aligned}$$

6. $\int x^2 \cdot 4^x dx$

Integration by parts: using tabular integration (or by parts twice), we have:

u	dv
x^2	4^x
$2x$	$\frac{1}{\ln 4}4^x$
2	$\frac{1}{(\ln 4)^2}4^x$
0	$\frac{1}{(\ln 4)^3}4^x$

So:

$$\int x^2 \cdot 4^x dx = \frac{1}{\ln 4}x^2 \cdot 4^x - \frac{2}{(\ln 4)^2}x \cdot 4^x + \frac{2}{(\ln 4)^3}4^x + C.$$