Math 1552, Integral Calculus
Section 10.7: Power Series

A **power series** centered at c is a *function* of x defined by: $f(x) = \sum_k a_k(x-c)^k$. Note that a power series is an infinite polynomial:

$$f(x) = \sum_k a_k(x-c)^k = a_0 + a_1(x-c) + a_2(x-c)^2 + a_3(x-c)^3 + ... + a_n(x-c)^n + ...$$

The **interval of convergence** is the interval of values for x that, when plugged into $f(x)$, result in a convergent series. The radius of convergence, R, is equal to half the length of the interval (i.e., to obtain the interval, we add and subtract R to the center). Don't forget to check the endpoints independently to test for convergence.

1. Consider the power series

$$f(x) = \sum_{k=1}^{\infty} \frac{2^k}{k+1}(x-3)^k.$$

a) What is $f(3)$? Can you generalize this statement: if $g(x) = \sum_k a_k(x-c)^k$, what is $g(c)$?

b) Write an expression for $f(4)$. Does this series converge or diverge? Use the convergence tests from class to justify your answer.

c) Write an expression for $f\left(\frac{5}{2}\right)$. Does this series converge absolutely, converge conditionally, or diverge?

d) In the power series above, $a_k = \frac{2^k}{k+1}$ is the **coefficient** term. Determine $L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$.

e) If L is finite and positive, we define the radius of convergence $R = \frac{1}{L}$. Find R.

f) We know from the ratio test that the series converges absolutely whenever $|x-c| < R$. Find an open interval on which our power series converges absolutely.

g) Since the ratio test is inconclusive when the limit equals 1, **we must hand check the endpoints**. Plug the endpoints into the function and determine if each series converges or diverges (hint: use your answer to part (c) to help!).

h) Summarize your results.