Math 1552: Integral Calculus

Review Problems for Test 1, Sections 5.1-5.6, 7.1-7.2

1. **Formula Recap**: complete each of the following formulas.

(a) The general Riemann Sum is found using the formula:

(b) Some helpful summation formulas are:

$$\sum_{i=1}^{n} c =$$

$$\sum_{i=1}^{n} i =$$

$$\sum_{i=1}^{n} i^2 =$$

(c) Properties of the definite integral:

$$\int_a^a f(x)dx =$$

$$\int_b^a f(x)dx =$$

$$\int_a^b cf(x)dx =$$

(d) State the Fundamental Theorem of Calculus:

(e) Using the FTC:

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)dt\right] =$$

(f) If F is an antiderivative of f, that means:

(g) If F is an antiderivative of f, then:

$$\int f(g(x))g'(x)dx =$$

$$\int_a^b f(g(x))g'(x)dx =$$

(h) To find the area between two curves, use the following steps:

(i) A *separable differential equation* has the general form:

To solve this equation, use the following steps:

2. Fill in the integration formulas below:

$$\int x^n dx, \quad (n \neq -1) =$$

$$\int \sin(ax)dx =$$

$$\int \cos(ax)dx =$$

$$\int \sec^2(ax)dx =$$

$$\int \sec(ax)\tan(ax)dx =$$

$$\int \csc(ax)\cot(ax)dx =$$

$$\int \csc^2(ax)dx =$$

$$\int \frac{1}{1+(ax)^2}dx =$$

$$\int \frac{1}{\sqrt{1-(ax)^2}}dx =$$

$$\int \frac{1}{x}dx =$$

$$\int e^{ax}dx =$$

$$\int b^{ax}dx =$$

$$\int \tan x dx =$$

$$\int \sec x dx =$$

$$\int \csc x dx =$$

$$\int \cot x dx =$$

3. (*Applying the Riemann Sum*) You are driving when all of a sudden, you see traffic stopped in front of you. You slam the brakes to come to a stop. While your brakes are applied, the velocity of the car is measured, and you obtain the following measurements:

Time since applying breaks (sec)	0	1	2	3	4	5
Velocity of car (in ft/sec)	88	60	40	25	10	0

Using the points given, determine upper and lower bounds for the total distance traveled before the car came to a stop.

4. Consider the function $f(x) = x + 2x^2$ on the interval $[0, 2]$. Using a midpoint estimate with $n = 4$ subintervals, estimate the *average value* of f.

5. (*Applying the Definite Integral*) A marketing company is trying a new campaign. The campaign lasts for three weeks, and during this time, the company finds that it gains customers as a function of time according to the formula:

$$C(t) = 5t - t^2,$$

where t is time in weeks and the number of customers is given in thousands.
Using the general form of the definite integral,

$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(x_i^*),$$

calculate the **average** number of customers gained during the three-week campaign.

6. Explain why the following property is true:

$$|\int_a^b f(x)dx| \leq \int_a^b |f(x)|dx.$$

Can you find an example where the inequality is strict?

7. Evaluate $\int_0^2 |x-1|dx$ using integral properties from class (you may use geometry, or a Riemann Sum).

8. Evaluate the integrals:

(a) $\int_1^2 \frac{3x-5}{x^3} dx$.

(b) $\int_1^3 |x-2| dx$.

9. Find $F'(2)$ for the function

$$F(x) = \int_{\frac{8}{x}}^{x^2} \left(\frac{t}{1-\sqrt{t}} \right) dt.$$

10. Evaluate the integrals:

$$\int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$$

$$\int \frac{\sin 3x - \cos 3x}{\sin 3x + \cos 3x} dx$$

$$\int \frac{1}{\ln(x^x)} dx$$

11. Evaluate the following integrals:

$$\int \frac{e^{2x}}{\sqrt{4-3e^{2x}}} dx$$

$$\int \frac{dx}{\sqrt{4-(x+3)^2}}$$

$$\int \frac{5p^{\sqrt{x+1}}}{\sqrt{x+1}} dx, \quad p > 0$$

12. Solve the initial value problem:

$$y' = x\sqrt{\frac{1-y^2}{1-x^2}}, \quad y(0) = 0.$$

13. Twenty percent of the candy in your Halloween bucket disappears after one hour. If the candy is disappearing exponentially, determine:

(a) the percent of candy after three hours, and

(b) the amount of time it will take until you have less than one percent of your candy left.

14. Find the area bounded between the curves $y = 2\cos x$ and $y = \sin(2x)$ on the interval $[-\pi, \pi]$.

15. Evaluate the integrals:

(a) $\int \left(\sqrt{x} - \frac{1}{x^2}\right)^2 dx$

(b) $\int \frac{\log_3 x^4}{x} dx$

(c) $\int \frac{\sec(e^{-4x})}{e^{4x}} dx$

(d) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$

(e) $\int \frac{x^2}{(ax^3+b)^2} dx$

(f) $\int_{-5}^{0} \left(x\sqrt{4-x}\right) dx$

(g) $\int (1 + \ln x) \cot(x \ln x) dx$

16. The velocity of a particle is given by the formula $v(t) = 5t^2 + 3t - 6$, in meters per second.

(a) Evaluate the actual distance traveled between time $t = 1$ and $t = 3$ seconds by dividing the interval into n equal subintervals and taking a limit of Riemann Sums, where x_i^* is chosen to be the right-hand endpoint of each subinterval.

(b) Check your answer to part (a) by calculating the actual value of $\int_1^3 v(t)dt$ using the FTC.

17. Find the general solution to the equation:

$$(y \ln x)y' = \frac{y^2+1}{x}.$$

18. Let $f(x) = 2x - x^2$.

(a) Sketch the graph of $f(x)$.

(b) Suppose we want to estimate $\int_0^1 f(x)dx$ using a general Riemann sum with n rectangles and x_i^* equal to the left endpoint of the ith subinterval. Would this be an overestimate or an underestimate of the true value of the integral? What if we chose x_i^* to be the right endpoint of the ith subinterval?

(c) How would your answers to (b) change if instead we wanted to approximate $\int_1^2 f(x)dx$?

19. Find the area bounded by the curves $y = -x^3 - 2x^2 + 7x - 2$ and $y = -x - 2$.

20. Evaluate the following integrals.

(a) $\int_0^{\ln(3)} e^{-2x} dx$

(b) $\int_0^8 \frac{x}{\sqrt{1+x}} dx$

21. Find the area bounded by the region between the curves $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$.

22. Find the area bounded by the region enclosed by the three curves $y = x^3$, $y = -x$, and $y = -1$.

23. Find the area of the triangle with vertices at the points (0,1), (3,4), and (4,2). USE CALCULUS.

24. As you go trick-or-treating, you walk with a velocity of $v(t) = \frac{e^t}{1+e^t}$ feet per minute. How far have you traveled after $\ln 5$ minutes?

25. Evaluate the definite integral $\int_0^{\pi/4} \left(\frac{\sec x}{\tan x + 1}\right)^2 dx$.

26. Calculuate $\frac{d}{dx}\left[\int_{x^2}^{x^4+1} e^{v^2} dv\right]$.

27. (a) Suppose that $f(x)$ is an even function such that $\int_{-1}^{1} f(x)dx = 1$ and $\int_0^3 f(x)dx = 4$. Find the value of $\int_1^3 f(x)dx$.

(b) Suppose that $g(x)$ is an odd function such that $\int_0^1 g(x)dx = 1$ and $\int_0^3 g(x)dx = 4$. Find the value of $\int_{-1}^{3} g(x)dx$.

28. Find the general solution to the equation

$$y\prime = \frac{\tan\left(y^2\right)}{y + xy}.$$

29. Integrate both sides of the double angle rule for sines to obtain a double angle rule for cosines.

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Answers

3. Upper bound is 223 ft; Lower bound is 135 ft

4. Midpoint estimate is 7.25 units^2, so the average value is approximately $3\frac{5}{8}$

5. 4,500 customers

6. The quantities represent absolute net area and total area. An example where the inequality is strict would be to consider $f(x) = x$ on the interval $[-1, 2]$.

7. 1

8. (a) $-\frac{3}{8}$, (b) 1

9. -24

10. (a) $-\sec\left(\frac{1}{x}\right) + C$

(b) $-\frac{1}{3}\ln|\sin 3x + \cos 3x| + C$

(c) $\ln|\ln x| + C$

11. (a) $-\frac{1}{3}\sqrt{4 - 3e^{2x}}$

(b) $\sin^{-1}\left(\frac{x+3}{2}\right) + C$

(c) $\frac{10}{\ln p} p^{\sqrt{x+1}} + C$

12. $y = \sin(-\sqrt{1 - x^2} + C)$.

13. (a) 51.2%, (b) $t = \frac{\ln(0.01)}{\ln(0.8)} \approx 20.64$ hours

14. 8 square units

15. (a) $\frac{1}{2}x^2 + \frac{4}{\sqrt{x}} - \frac{1}{3x^3} + C$

(b) $2\ln 3(\log_3 x)^2 + C$

(c) $-\frac{1}{4}\ln|\sec(e^{-4x}) + \tan(e^{-4x})| + C$

(d) $\frac{2}{3}$

(e) $-\frac{1}{3a(ax^3+b)} + C$

(f) $-\frac{506}{15}$

(g) $\ln|\sin(x\ln x)| + C$

16. $\frac{130}{3}$ square units

17. $y^2 = k(\ln x)^2 - 1$, where $k = e^{C^*}$

18. (a) a parabola pointing downward with vertex at $(1, 1)$

(b) left-hand endpoint is an underestimate; right-hand endpoint is an overestimate

(c) the answers to (b) will reverse

19. -36

20. (a) $\frac{4}{9}$, (b) $\frac{40}{3}$

21. $\frac{37}{12}$ square units

22. $\frac{5}{4}$ square units

23. 4.5 square units

24. $\ln 3$ feet

25. $\frac{1}{2}$

26. $4x^3e^{\left(x^4+1\right)^2} - 2xe^{\left(x^2\right)^2}$

27. (a) 3.5 (b) 3

28. $y^2 = \sin^{-1}\left[C\left(1+x\right)^2\right]$, $C \neq 0$

29. $\cos\left(2\theta\right) = 1 - 2\sin^2\left(\theta\right)$ or $\cos\left(2\theta\right) = 1 + 2\cos^2\left(\theta\right)$