Math 1552: Integral Calculus

Review Problems for Test 1, Sections 4.8, 5.1-5.6

1. **Formula Recap**: complete each of the following formulas.

(a) The general Riemann Sum is found using the formula:

(b) Some helpful summation formulas are:

$$\sum_{i=1}^{n} c =$$

$$\sum_{i=1}^{n} i =$$

$$\sum_{i=1}^{n} i^2 =$$

(c) Properties of the definite integral:

$$\int_a^a f(x)dx =$$

$$\int_b^a f(x)dx =$$

$$\int_a^b cf(x)dx =$$

(d) State the Fundamental Theorem of Calculus:

(e) Using the FTC:

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)dt\right] =$$

(f) If F is an antiderivative of f, that means:

(g) If F is an antiderivative of f, then:

$$\int f(g(x))g'(x)dx =$$

$$\int_a^b f(g(x))g'(x)dx =$$

(h) To find the area between two curves, use the following steps:

2. Fill in the integration formulas below:

$$\int x^n dx, \quad (n \neq -1) =$$

$$\int \sin(ax)dx =$$

$$\int \cos(ax)dx =$$

$$\int \sec^2(ax)dx =$$

$$\int \sec(ax)\tan(ax)dx =$$

$$\int \csc(ax)\cot(ax)dx =$$

$$\int \csc^2(ax)dx =$$

$$\int \frac{1}{1+(ax)^2}dx =$$

$$\int \frac{1}{\sqrt{1-(ax)^2}}dx =$$

$$\int \frac{1}{x}dx =$$

$$\int e^{ax}dx =$$

$$\int b^{ax}dx =$$

$$\int \tan x dx =$$

$$\int \sec x dx =$$

$$\int \csc x dx =$$

$$\int \cot x dx =$$

3. (*Applying the Riemann Sum*) You are driving when all of a sudden, you see traffic stopped in front of you. You slam the brakes to come to a stop. While your brakes are applied, the velocity of the car is measured, and you obtain the following measurements:

Time since applying breaks (sec)	0	1	2	3	4	5
Velocity of car (in ft/sec)	88	60	40	25	10	0

Using the points given, determine upper and lower bounds for the total distance traveled before the car came to a stop.

4. Consider the function $f(x) = x + 2x^2$ on the interval $[0, 2]$. Using a midpoint estimate with $n = 4$ subintervals, estimate the *average value* of f.

5. Evaluate $\int_0^2 |x - 1|dx$ using integral properties from class (you may use geometry, or a Riemann Sum).

6. Evaluate the integrals:

(a) $\int_1^2 \frac{3x-5}{x^3}dx$.

(b) $\int_1^3 |x - 2|dx$.

7. (*Applying the Definite Integral*) A marketing company is trying a new campaign. The campaign lasts for three weeks, and during this time, the company finds that it gains customers as a function of time according to the formula:

$$C(t) = 5t - t^2,$$

where t is time in weeks and the number of customers is given in thousands.

Using the general form of the definite integral,

$$\int_a^b f(x)dx = \lim_{n\to\infty} \frac{b-a}{n} \sum_{i=1}^{n} f(x_i^*),$$

calculate the **average** number of customers gained during the three-week campaign.

8. Find $F'(2)$ for the function

$$F(x) = \int_{\frac{8}{x}}^{x^2} \left(\frac{t}{1-\sqrt{t}}\right) dt.$$

9. Evaluate the integrals:

$$\int \frac{1}{x^2} \sec\left(\frac{1}{x}\right) \tan\left(\frac{1}{x}\right) dx$$

$$\int \frac{\sin 3x - \cos 3x}{\sin 3x + \cos 3x} dx$$

$$\int \frac{1}{\ln(x^x)} dx$$

10. Evaluate the following integrals:

$$\int \frac{e^{2x}}{\sqrt{4-3e^{2x}}} dx$$

$$\int \frac{dx}{\sqrt{4-(x+3)^2}}$$

$$\int \frac{5p^{\sqrt{x+1}}}{\sqrt{x+1}} dx, \quad p > 0$$

11. Evaluate the integrals:

(a) $\int \left(\sqrt{x} - \frac{1}{x^2}\right)^2 dx$

(b) $\int \frac{\log_3 x^4}{x} dx$

(c) $\int \frac{\sec(e^{-4x})}{e^{4x}} dx$

(d) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$

(e) $\int \frac{x^2}{(ax^3+b)^2} dx$

(f) $\int_{-5}^0 \left(x\sqrt{4-x}\right) dx$

(g) $\int (1+\ln x)\cot(x\ln x)dx$

12. The velocity of a particle is given by the formula $v(t) = 5t^2 + 3t - 6$, in meters per second.

(a) Evaluate the actual distance traveled between time $t = 1$ and $t = 3$ seconds by dividing the interval into n equal subintervals and taking a limit of Riemann Sums, where x_i^* is chosen to be the right-hand endpoint of each subinterval.

(b) Check your answer to part (a) by calculating the actual value of $\int_1^3 v(t)dt$ using the FTC.

13. Find the area bounded by the curves $y = -x^3 - 2x^2 + 7x - 2$ and $y = -x - 2$.

14. Find the area bounded by the region between the curves $f(x) = x^3 + 2x^2$ and $g(x) = x^2 + 2x$.

15. Find the area bounded by the region enclosed by the three curves $y = x^3$, $y = -x$, and $y = -1$.

16. Find the area of the triangle with vertices at the points (0,1), (3,4), and (4,2). USE CALCULUS.

17. Use the Fundamental Theorem of Calculus to evaluate the definite integral:

$$\int_{\pi/2}^{\pi} \left(\frac{1}{x^2} + 2\sqrt{x} + \cos x\right) dx.$$

18. Let $y = \int_{-4}^{\tan x} \sin(t^2)dt$. Find $\frac{dy}{dx}$ for $0 < x < \pi/4$.

19. Using the general form of the definite integral, $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i^*)\Delta x$, evaluate: $\int_2^4 (x-1)^2 dx$. Then check your answer using the Fundamental Theorem of Calculus.

20. Use the midpoint rule with 4 subintervals to approximate the average of the function $f(t) = \frac{1}{2} + \sin^2(\pi t)$ on the interval $[0, 2]$.

21. Give an upper bound and a lower bound for the area under the curve $y = x^2$ over the interval $[0, 2]$ by partitioning into four subintervals.

22. Write down a summation that estimates the average value of the function $f(x) = e^{x^2}$ on the interval $[1, 3]$ using 2018 intervals of equal width and left-endpoints. You do not need to simplify.

23. Express the following limit as a definite integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{9}{n} \log\left(2\left(1+\frac{9i}{n}\right)^2\right)$$

24. Let t be a positive real number. Evaluate the integral $\int_{-t}^{t} x^2 dx$ as a limit of Riemann sums. Your answer should be in terms of t. Show work. Do not use the Fundamental Theorem.

25. Evaluate the following definite integrals. Show work and briefly explain your answers.

(a) $\int_{-3}^{3} \sqrt{9-x^2}dx$

(b) $\int_{-3}^{3} \sin(x^5)dx$

(c) $\int_{-1}^{2} |2x|dx$

26. Evaluate the following integrals.

(a) $\int_0^{\frac{\pi}{3}} \cos(3x)\sin(3x)dx$

(b) $\int (6\csc(4x)\cot(4x) - 3\sqrt{x} + 1)\, dx$

(c) $\int \frac{6}{\sqrt{y}(5+6\sqrt{y})^5} dy$

(d) $\int \frac{1}{\sqrt{x}e^{-\sqrt{x}}} \csc^2\left(3e^{\sqrt{x}} - 3\right) dx$

(e) $\int \frac{1}{\sin^{-1}(x)\sqrt{1-x^2}} dx$

27. Find the limit of the Riemann sum by first writing the limit as a definite integral and then performing an appropriate u-substitution to evaluate the definite integral.

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{n} \left(\left(3+\frac{i}{n}\right)\left(\left(3+\frac{i}{n}\right)^2 - 10\right)^4\right)$$

28. Solve the separable differential equation. Write your answer $y = f(x)$ a function of x, with an arbitrary constant "C" in the correct place.

$$\frac{dy}{dx} = x^2\sqrt{y}, \quad y > 0$$

29. Solve the separable differential equation by first separating the variables and writing the differential equation as $\frac{dy}{dx} = f(x)g(y)$.

$$\sec x \frac{dy}{dx} = e^{y+\sin x}$$

Answers

3. Upper bound is 223 ft; Lower bound is 135 ft

4. Midpoint estimate is 7.25 units^2, so the average value is approximately $3\frac{5}{8}$

5. 1

6. (a) $-\frac{3}{8}$, (b) 1

7. 4,500 customers

8. -24

9. (a) $-\sec\left(\frac{1}{x}\right) + C$

(b) $-\frac{1}{3}\ln|\sin 3x + \cos 3x| + C$

(c) $\ln|\ln x| + C$

10. (a) $-\frac{1}{3}\sqrt{4 - 3e^{2x}}$

(b) $\sin^{-1}\left(\frac{x+3}{2}\right) + C$

(c) $\frac{10}{\ln p}p^{\sqrt{x+1}} + C$

11. (a) $\frac{1}{2}x^2 + \frac{4}{\sqrt{x}} - \frac{1}{3x^3} + C$

(b) $2\ln 3(\log_3 x)^2 + C$

(c) $-\frac{1}{4}\ln|\sec(e^{-4x}) + \tan(e^{-4x})| + C$

(d) $\frac{2}{3}$

(e) $-\frac{1}{3a(ax^3+b)} + C$

(f) $-\frac{506}{15}$

(g) $\ln|\sin(x \ln x)| + C$

12. $\frac{130}{3}$ square units

13. $\frac{148}{3}$

14. $\frac{37}{12}$ square units

15. $\frac{5}{4}$ square units

16. 4.5 square units

17. $-\frac{1}{\pi}+4/3\sqrt{\pi^3}-\left(-\frac{1}{\pi/2}+4/3\sqrt{(\pi/2)^3}+1\right)$

18. $\sin\left(\tan^2 x\right)\cdot\sec^2 x$

19. $\frac{26}{3}$

20. 1

21. Upper Bound$=\frac{15}{4}$, Lower Bound$=\frac{7}{4}$

22. $\frac{1}{2}\cdot\frac{2}{2018}\cdot\sum_{i=0}^{2017}e^{(1+\frac{2i}{2018})^2}$ or $\frac{1}{2}\cdot\frac{2}{2018}\cdot\sum_{i=1}^{2018}e^{(1+\frac{2(i-1)}{2018})^2}$

23. $\int_1^{10}\log(2x^2)dx$

24. $\frac{2}{3}t^3$

25. (a) $\frac{9}{2}\pi$, (b) 0, (c) 5

26. (a) 0

(b) $-\frac{3}{2}\csc(4x)-2x^{3/2}+x+C$

(c) $-\frac{1}{2(5+6\sqrt{y})^4}+C$

(d) $-\frac{2}{3}\cot\left(3e^{\sqrt{x}}-3\right)+C$

(c) $\ln\left(\sin^{-1}(x)\right)+C$

27. 777.7

28. $y=(\frac{1}{6}x^3+C)^2$

29. $y=-\ln(-e^{-\sin x}+C)$