Math 1552, Integral Calculus

Review for Test 2

Sections 8.2-8.5, 4.5, 8.8

1. Content Recap

(a) To apply L'Hopital's rule, the limit must have the indeterminate form _____________ or _____________.

(b) An integral $\int_a^b f(x)dx$ is *improper* if at least one of the limits of integration is _____________, or if there is a _____________ _____________ on the interval $[a, b]$.

(c) Evaluate an integral using *integration by parts* if:

To choose the value of u, use the rule: _____________.

(d) To evaluate integrals with powers or products of trig functions, use the following trig identities to try to obtain a u-substitution:

(e) If we would evaluate an integral using *trig substitution*, the integral should contain an expression of one of these forms: _____________, _____________, or _____________.

Write out the trig substitution you would use for each form listed above.

(f) To use the method of *partial fractions*, we must first factor the denominator completely into ___________ or ___________ ___________ terms.

In the partial fraction decomposition, if the term in the denominator is raised to the kth power, then we have ___________ partial fractions.

For each linear term, the numerator of the partial fraction will be ___________.

For each irreducible quadratic term, the numerator will be ___________.

2. Evaluate the following limits:

$$\lim_{x \to 0} \frac{\ln(\sec x)}{x^2}, \quad \lim_{x \to \infty} \left[\cos\left(\frac{1}{x}\right)\right]^x.$$

3. Evaluate each integral below using any of the methods we have learned.

(a) $\int \frac{\sin^3 x}{\cos x} dx$

(b) $\int \frac{x}{\sqrt{x^2+2x-3}} dx$

(c) $\int \frac{\cos x}{4+\sin^2 x} dx$

(d) $\int \frac{1}{x(x^2+x+1)} dx$

4. Evaluate the improper integral if it converges, or show that the integral diverges.

$$\int_0^3 \frac{x}{(x^2-1)^{2/3}} dx$$

5. Does the integral

$$\int_0^\infty \frac{dx}{e^x + e^{-x}}$$

converge or diverge? (HINT: Use the Integral Comparison Test.)

6. Evaluate the following integrals.

(a) $\int 3x \cos(2x) dx$

(b) $\int x^5 \ln(x) dx$

(c) $\int x^3 e^{x^2} dx$

(d) $\int (\ln x)^2 dx$

(e) $\int x^2 \cdot 4^x dx$

(f) $\int \cos(2x)e^x dx$

7. Evaluate the following integrals using any of the integration techniques we have learned.

(a) $\int \sin^5(2x)\cos^3(2x)dx$

(b) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$

(c) $\int \tan^4(x)dx$

8. Evaluate the following integrals using any method we have learned so far.

(a) $\int \frac{x^2}{(x^2+4)^{3/2}} dx$

(b) $\int \frac{\sqrt{1-x^2}}{x^4} dx$

(c) $\int \frac{x}{(4-x^2)^{3/2}} dx$

(d) $\int \frac{dx}{e^x\sqrt{e^{2x}-9}}$

(e) $\int \sin^2(x)\cos^2(x)dx$

(f) $\int (x^2+1)e^{2x}dx$

9. Evaluate the following integrals using any method we have learned.

(a) $\int \frac{x+3}{(x-1)(x^2-4x+4)} dx$

(b) $\int \frac{x+4}{x^3+x} dx$

(c) $\int \tan(x)\ln[\cos(x)]dx$

(d) $\int \frac{x+2}{x+1} dx$

(e) $\int \sqrt{25-x^2}dx$

(f) $\int \tan^3(x)\sec^4(x)dx$

(g) $\int x\tan^{-1}(x)dx$

(h) $\int \frac{dx}{x\sqrt{1+x^2}}$

(i) $\int \frac{x+1}{x^2(x-1)} dx$

(j) $\int \frac{x+1}{x^2-4x+8} dx$

10. Let θ be an angle in radians, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, so that $\tan\theta = \frac{1}{2}$.

We can find the value of θ using the integral:

$$\theta = \tan^{-1}\left(\frac{1}{2}\right) = \int_0^{1/2} \frac{1}{1+x^2}\,dx.$$

(a) Estimate the value of θ with the trapezoidal rule using $n = 4$ subintervals.

(b) Using the formula for error in the Trapezoidal rule, estimate the largest possible error for your answer to problem 1.

(c) The actual value is approximately 0.46365. What is the actual error, and the percent error in your estimate in problem 1?

(d) Estimate the value of θ with Simpson's rule using $n = 6$ subintervals.

11. Evaluate the following limits using L'Hopital's Rule.

(a) $\lim_{x\to 0}\left[\frac{1}{x} - \cot x\right]$

(b) $\lim_{x\to 0^+}\left[x(\ln(x))^2\right]$

(c) $\lim_{x\to\infty}(x + e^x)^{2/x}$

(d) $\lim_{x\to\frac{\pi}{2}}\left[\frac{\ln(\sin x)}{(\pi-2x)^2}\right]$

12. Find values of a and b so that

$$\lim_{x\to 0}\frac{\cos(ax) - b}{2x^2} = -4.$$

13. Evaluate the improper integrals if they converge, or show that the integral diverges.

$$\int_1^3 \frac{1}{(x^2-1)^{3/2}}\,dx$$

$$\int_0^\infty x^2 e^{-2x}\,dx$$

14. For what values of p does the integral $\int_4^\infty \frac{dx}{x(\ln x)^p}$ converge?

15. Find the area bounded by the curve $y = \frac{1}{x^2+9}$, the x-axis, and $x \geq 0$.

16. Evaluate each indefinite integral.

(a) $\int \frac{1}{x^2\sqrt{1+x^2}}dx$

(b) $\int \frac{x^5}{\sqrt{4-x^2}}dx$

17. Evaluate the following limits using L'Hopital's Rule.

(a) $\lim_{x\to 0^+} e^{-x\ln(x)}$

(b) $\lim_{x\to 0} \frac{x\tan^{-1}(x)}{1-\cos(x)}$

18. Either evaluate the integral or show that it diverges: $\int_1^\infty \frac{1-\ln(x)}{x^2}dx$.

19. For which values of p does the following integral converge? $\int_0^\infty \frac{dx}{x^p}$.

20. Evaluate the following integrals.

(a) $\int \tan^3(x)\sec^3 x dx$

(b) $\int \cot(x)\sec^2 x dx$

(c) $\int \frac{\sin^7(x)}{\cos^4(x)}dx$

(d) $\int \sec^4(x)dx$

21. Evaluate the following integrals.

(a) $\int \frac{x^3-1}{x^2+1}dx$

(b) $\int \frac{\cos(2x)}{\sin^2(2x)-3\sin(2x)-4}dx$

22. Find an upper bound on the error of approximating $\int_3^5 f(x)\ dx$, with $f(x) = \frac{1}{2x-4}$ using (a) Simpson's Rule, and (b) the Trapezoid Rule, with $n = 4$ subintervals. Use the formulas

$$|E_S| \leq \frac{M(b-a)^5}{180n^4}; \qquad M \geq |f^{(4)}(x)|, \quad x \in [a,b],$$

and

$$|E_T| \leq \frac{M(b-a)^3}{12n^2}; \qquad M \geq |f''(x)|, \quad x \in [a,b].$$

23. Repeat problem #22 with $f(x) = \frac{-1}{2x-4}$.

ANSWERS

2. $\frac{1}{2}$, 1

3. (a) $-\ln|\cos x| + \frac{1}{2}\cos^2 x + C$

(b) $\sqrt{x^2+2x-3} - \ln|\frac{x+1+\sqrt{x^2+2x-3}}{2}| + C$

(c) $\frac{1}{2}\tan^{-1}\left(\frac{\sin x}{2}\right) + C$

(d) $\ln|x| - \frac{1}{2}\ln(x^2+x+1) - \frac{\sqrt{3}}{3}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + C$

4. Converges to $\frac{9}{2}$

5. Converges (compare to $\int_0^\infty \frac{1}{e^x}dx$)

6. (a) $\frac{3}{2}\sin(2x) + \frac{3}{4}\cos(2x) + C$

(b) $\frac{x^6 \ln x}{6} - \frac{x^6}{36} + C$

(c) $\frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2} + C$

(d) $x(\ln x)^2 - 2x\ln x + 2x + C$

(e) $\frac{1}{\ln 4}x^2 \cdot 4^x - \frac{2}{(\ln 4)^2}x \cdot 4^x + \frac{2}{(\ln 4)^3}4^x + C$

(f) $\frac{1}{5}\cos(2x)e^x + \frac{2}{5}\sin(2x)e^x + C$

7. (a) $\frac{1}{12}\sin^6(2x) - \frac{1}{16}\sin^8(2x) + C$

(b) $\frac{2}{3}$

(c) $\frac{1}{3}\tan^3(x) - \tan(x) + x + C$

8. (a) $\ln|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}| - \frac{x}{\sqrt{x^2+4}} + C$

(b) $-\frac{1}{3} \cdot \frac{(1-x^2)^{3/2}}{x^3} + C$

(c) $\frac{1}{\sqrt{4-x^2}} + C$

(d) $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$

(e) $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$

(f) $\frac{1}{2}(x^2+1)e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

9. (a) $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$

(b) $4\ln|x| - 2\ln(x^2+1) + \tan^{-1}(x) + C$

(c) $-\frac{1}{2}(\ln[\cos(x)])^2 + C$

(d) $x + \ln|x+1| + C$

(e) $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)+\frac{x\sqrt{25-x^2}}{2}+C$

(f) $\frac{1}{4}\tan^4(x)+\frac{1}{6}\tan^6(x)+C$

(g) $\frac{x^2}{2}\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+C$

(h) $-\ln\left|\frac{\sqrt{1+x^2}}{x}+\frac{1}{x}\right|+C$

(i) $-2\ln|x|+\frac{1}{x}+2\ln|x-1|+C$

(j) $\frac{1}{2}\ln|x^2-4x+8|+\frac{3}{2}\tan^{-1}\left(\frac{x-2}{2}\right)+C$

10. (a) ≈ 0.46281, (b) $\frac{1}{768}\approx 0.0013$, (c) 0.00084, (d) ≈ 0.46365

11. (a) 0, (b) 0, (c) e^2, (d) $-\frac{1}{8}$

12. $a=\pm 4$, $b=1$

13. (a) diverges, (b) converges to $\frac{1}{4}$

14. converges when $p>1$

15. $\frac{\pi}{6}$ units^2

16. (a) $\frac{\sqrt{1+x^2}}{x}+C$

(b) $-32\left(\left(\frac{\sqrt{4-x^2}}{2}\right)-\frac{2}{3}\left(\frac{\sqrt{4-x^2}}{2}\right)^3+\frac{1}{5}\left(\frac{\sqrt{4-x^2}}{2}\right)^5\right)+C$

17. (a) 1, (b) 2

18. 0

19. Diverges for all p

20. (a) $-\frac{1}{3}\sec^3(x)+\frac{1}{5}\sec^5(x)+C$
(b) $\ln(\tan(x))+C$
(c) $\frac{1}{3\cos^3(x)}-\frac{3}{\cos(x)}-3\cos(x)+\frac{1}{3}\cos^3(x)+C$
(d) $(2\tan(x))/3+1/3\sec(x)^2\tan(x)+C$

21. (a) $\frac{1}{2}x^2-\frac{1}{2}\ln(x^2+1)-\tan^{-1}(x)+C$

(b) $-\frac{1}{10}\ln|\sin(2x)+1|+\frac{1}{10}\ln|\sin(2x)-4|+C$

22. (a) $\frac{1}{120}$, (b) $\frac{1}{24}$

23. (a) $\frac{1}{29{,}160}$, (b) $\frac{1}{648}$