Math 1552, Integral Calculus

Review for Test 2

Sections 7.2, 8.2-8.5, 4.5, 8.8

1. Content Recap

(a) A *separable differential equation* has the general form:

To solve this equation, use the following steps:

(b) To apply L'Hopital's rule, the limit must have the indeterminate form _____ or _____.

(c) An integral $\int_a^b f(x)dx$ is *improper* if at least one of the limits of integration is _____, or if there is a _____ _____ on the interval $[a, b]$.

(d) Evaluate an integral using *integration by parts* if:

To choose the value of u, use the rule: _____.

(e) To evaluate integrals with powers or products of trig functions, use the following trig identities to try to obtain a u-substitution:

(f) If we would evaluate an integral using *trig substitution*, the integral should contain an expression of one of these forms: _____, _____, or _____.

Write out the trig substitution you would use for each form listed above.

(g) To use the method of *partial fractions*, we must first factor the denominator completely into _____ or _____ _____ terms.

In the partial fraction decomposition, if the term in the denominator is raised to the kth power, then we have _____ partial fractions.

For each linear term, the numerator of the partial fraction will be _____.

For each irreducible quadratic term, the numerator will be _____.

2. Evaluate the following limits:

$$\lim_{x\to 0}\frac{\ln(\sec x)}{x^2}, \quad \lim_{x\to\infty}\left[\cos\left(\frac{1}{x}\right)\right]^x.$$

3. Evaluate each integral below using any of the methods we have learned.

(a) $\int \frac{\sin^3 x}{\cos x}dx$

(b) $\int \frac{\cos x}{4+\sin^2 x}dx$

4. Evaluate the improper integral if it converges, or show that the integral diverges.

$$\int_0^3 \frac{x}{(x^2-1)^{2/3}}dx$$

5. Solve the initial value problem $y' = xy - x - y + 1$, $y(1) = 0$.

6. Evaluate the following integrals.

(a) $\int 3x\cos(2x)dx$

(b) $\int x^5\ln(x)dx$

(c) $\int x^3 e^{x^2}dx$

(d) $\int(\ln x)^2 dx$

(e) $\int x^2 \cdot 4^x dx$

(f) $\int \cos(2x) e^x dx$

7. Evaluate the following integrals using any of the integration techniques we have learned.

(a) $\int \sin^5(2x) \cos^3(2x) dx$

(b) $\int_1^e \frac{\sqrt{\ln x}}{x} dx$

(c) $\int \tan^4(x) dx$

(d) $\int \sqrt{x} e^{\sqrt{x}} dx$

(e) $\int_0^{\pi/2} (\sin x)^{11} dx$

8. Evaluate the following integrals using any method we have learned so far.

(a) $\int \frac{x^2}{(x^2+4)^{3/2}} dx$

(b) $\int \frac{\sqrt{1-x^2}}{x^4} dx$

(c) $\int \frac{x}{(4-x^2)^{3/2}} dx$

(d) $\int \frac{dx}{e^x \sqrt{e^{2x}-9}}$

(e) $\int \sin^2(x) \cos^2(x) dx$

(f) $\int (x^2+1) e^{2x} dx$

9. Evaluate the following integrals using any method we have learned.

(a) $\int \frac{x+3}{(x-1)(x^2-4x+4)} dx$

(b) $\int \frac{x+4}{x^3+x} dx$

(c) $\int \tan(x) \ln[\cos(x)] dx$

(d) $\int \frac{x+2}{x+1} dx$

(e) $\int \sqrt{25-x^2} dx$

(f) $\int \tan^3(x) \sec^4(x) dx$

(g) $\int x \tan^{-1}(x) dx$

(h) $\int \frac{dx}{x\sqrt{1+x^2}}$

(i) $\int \frac{x+1}{x^2(x-1)} dx$

10. At a bank, interest is compounded continuously, that is, the rate of change is equal to the account value times the interest rate. The interest rate is changing and is given at time t years by the formula $0.05+0.015\sqrt{t}$. Suppose a person deposits \$1000 in an account at time $t = 0$ and leaves it alone.

(a) Set up and solve a differential equation to get a formula for the amount in the account at time t.

(b) Find the amount in the account after 2 years.

11. Evaluate the following limits using L'Hopital's Rule.

(a) $\lim_{x\to 0}\left[\frac{1}{x} - \cot x\right]$

(b) $\lim_{x\to 0^+}[x(\ln(x))^2]$

(c) $\lim_{x\to\infty}(x + e^x)^{2/x}$

(d) $\lim_{x\to\frac{\pi}{2}}\left[\frac{\ln(\sin x)}{(\pi-2x)^2}\right]$

12. Find values of a and b so that

$$\lim_{x\to 0}\frac{\cos(ax) - b}{2x^2} = -4.$$

13. Evaluate the improper integrals if they converge, or show that the integral diverges.

$$\int_1^3 \frac{1}{(x^2-1)^{3/2}}dx$$

$$\int_0^\infty x^2 e^{-2x}dx$$

14. For what values of p does the integral $\int_4^\infty \frac{dx}{x(\ln x)^p}$ converge?

15. Find the area bounded by the curve $y = \frac{1}{x^2+9}$, the x-axis, and $x \geq 0$.

16. (a) Find the partial fractions decomposition for the function $f(x) = \frac{1}{x^2(4+x^2)}$.

(b) Use your answer to part (a) to evaluate $\int f(x)dx$.

(c) Could you use an alternate method to evaluate the integral in part (b)? If yes, which method?

17. Evaluate the following integrals.

(a) $\int \frac{x^2}{(x-1)(x^2+2x+1)}\,dx$

(b) $\int \frac{1}{(x+1)(x^2+1)}\ dx$

(c) $\int \frac{x^3}{x^2-2x+1}\ dx$

18. Either evaluate the integral or show that it diverges: $\int_1^\infty \frac{1-\ln(x)}{x^2}dx$.

19. Evaluate the following integrals using trigonometric substitutions.

(a) $\int \frac{\sqrt{1-x^2}}{x^2}\ dx$

(b) $\int \frac{1}{\sqrt{(1+x^2)^3}}\ dx$

20. Evaluate the following integrals.

(a) $\int \tan^3(x)\sec^3(x)\ dx$

(b) $\int \cot(x)\sec^2(x)\ dx$

(c) $\int \frac{\sin^7(x)}{\cos^4(x)}dx$

(d) $\int \sec^4(x)\ dx$

21. Evaluate the following integrals.

(a) $\int \frac{x^3-1}{x^2+1}dx$

(b) $\int \frac{\cos(2x)}{\sin^2(2x)-3\sin(2x)-4}dx$

22. Evaluate the indefinite integrals.

(a) $\int \tan^5(x)dx$

(b) $\int \sin^7(x)dx$

(c) $\int \sin^4(x)dx$

ANSWERS

2. $\frac{1}{2}$, 1

3. (a) $-\ln|\cos x| + \frac{1}{2}\cos^2 x + C$
(b) $\frac{1}{2}\tan^{-1}\left(\frac{\sin x}{2}\right) + C$

4. Converges to $\frac{9}{2}$

5. $y = -e^{(x-1)^2/2} + 1$ or $y = -\sqrt{e}\cdot e^{x^2/2-x} + 1$

6. (a) $\frac{3}{2}x\sin(2x) + \frac{3}{4}\cos(2x) + C$
(b) $\frac{x^6\ln x}{6} - \frac{x^6}{36} + C$
(c) $\frac{x^2e^{x^2}}{2} - \frac{e^{x^2}}{2} + C$
(d) $x(\ln x)^2 - 2x\ln x + 2x + C$
(e) $\frac{1}{\ln 4}x^2\cdot 4^x - \frac{2}{(\ln 4)^2}x\cdot 4^x + \frac{2}{(\ln 4)^3}4^x + C$
(f) $\frac{1}{5}\cos(2x)e^x + \frac{2}{5}\sin(2x)e^x + C$

7. (a) $\frac{1}{12}\sin^6(2x) - \frac{1}{16}\sin^8(2x) + C$
(b) $\frac{2}{3}$
(c) $\frac{1}{3}\tan^3(x) - \tan(x) + x + C$
(d) $2e^{\sqrt{x}}(x - 2\sqrt{x} + 2)$
(e) $\frac{10\cdot 8\cdot 6\cdot 4\cdot 2}{11\cdot 9\cdot 7\cdot 5\cdot 3}$

8. (a) $\ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| - \frac{x}{\sqrt{x^2+4}} + C$

(b) $-\frac{1}{3}\cdot\frac{(1-x^2)^{3/2}}{x^3} + C$

(c) $\frac{1}{\sqrt{4-x^2}} + C$

(d) $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$

(e) $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$

(f) $\frac{1}{2}(x^2+1)e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

9. (a) $4\ln\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + C$

(b) $4\ln|x| - 2\ln(x^2+1) + \tan^{-1}(x) + C$

(c) $-\frac{1}{2}(\ln[\cos(x)])^2 + C$

(d) $x + \ln|x+1| + C$

(e) $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right)+\frac{x\sqrt{25-x^2}}{2}+C$

(f) $\frac{1}{4}\tan^4(x)+\frac{1}{6}\tan^6(x)+C$

(g) $\frac{x^2}{2}\tan^{-1}(x)-\frac{x}{2}+\frac{1}{2}\tan^{-1}(x)+C$

(h) $-\ln\left|\frac{\sqrt{1+x^2}}{x}+\frac{1}{x}\right|+C$

(i) $-2\ln|x|+\frac{1}{x}+2\ln|x-1|+C$

10. (a) $1000\,e^{0.05t+0.01t^{3/2}}$

(b) $1000\,e^{0.1+0.02\sqrt{2}}$ or \$1136.88

11. (a) 0, (b) 0, (c) e^2, (d) $-\frac{1}{8}$

12. $a=\pm 4$, $b=1$

13. (a) diverges, (b) converges to $\frac{1}{4}$

14. converges when $p>1$

15. $\frac{\pi}{6}$ units^2

16. (a) $\frac{1/4}{x^2}-\frac{1/4}{4+x^2}$

(b) $-\frac{1}{4x}-\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C$

(c) a trig sub would also work

17. (a) $\frac{1}{4}\ln|x-1|+\frac{3}{4}\ln|x+1|+\frac{1}{2(x+1)}+C$

(b) $\frac{1}{2}\ln|x+1|-\frac{1}{4}\ln(x^2+1)-\frac{1}{2}\tan^{-1}(x)+C$

(c) $\frac{x^2}{2}+2x+3\ln|x-1|-\frac{1}{x-1}+C$

18. 0

19. (a) $-\frac{\sqrt{1-x^2}}{x}-\sin^{-1}x+C$

(b) $\frac{x}{\sqrt{1+x^2}}+C$

20. (a) $-\frac{1}{3}\sec^3(x)+\frac{1}{5}\sec^5(x)+C$

(b) $\ln(\tan(x))+C$

(c) $\frac{1}{3\cos^3(x)}-\frac{3}{\cos(x)}-3\cos(x)+\frac{1}{3}\cos^3(x)+C$

(d) $(2\tan(x))/3+1/3\sec(x)^2\tan(x)+C$

21. (a) $\frac{1}{2}x^2-\frac{1}{2}\ln(x^2+1)-\tan^{-1}(x)+C$

(b) $-\frac{1}{10}\ln|\sin(2x)+1|+\frac{1}{10}\ln|\sin(2x)-4|+C$

22. (a) $\int \ln|\sec(x)| + \frac{1}{4}\sec^4(x) - \sec^2(x) + C$

(b) $\frac{1}{7}\cos^7(x) - \frac{3}{5}\cos^5(x) + \cos^3(x) - \cos^x + C$

(c) $\frac{1}{32}\sin(4x) - \frac{1}{4}\sin(2x) + \frac{x}{3} + C$