Math 1552, Integral Calculus

Test 3 Review, Sections 10.1-10.7

1. Terminology review: complete the following statements.

(a) A geometric series has the general form ____________.
The series converges when ____________ and diverges when ____________.

(b) A p-series has the general form ____________. The series converges when ____________ and diverges when ____________. To show these results, we can use the ____________ test.

(c) The harmonic series ____________ and telescoping series ____________.

(d) If you want to show a series converges, compare it to a ____________ series that also converges. If you want to show a series diverges, compare it to a ____________ series that also diverges.

(e) If the direct comparison test does not have the correct inequality, you can instead use the ____________ ____________ test. In this test, if the limit is a ____________ number (not equal to ____________), then both series converge or both series diverge.

(f) In the ratio and root tests, the series will ____________ if the limit is less than 1 and ____________ if the limit is greater than 1. If the limit equals 1, then the test is ____________.

(g) If $\lim_{n\to\infty} a_n = 0$, then what do we know about the series $\sum_n a_n$? ____________

(h) A power series has the general form: ____________. To find the radius of convergence R, use either the ____________ or ____________ test. The series converges ____________ when $|x - c| < R$. To find the interval of convergence, don't forget to check the ____________.

2. Let $\{a_n\}$ be a sequence of non-negative terms. Are the following statements *always* true or sometimes false?

(a) If $\lim_{n\to\infty} a_n = L$, then the series $\sum_n a_n = L$.

(b) If $\{a_n\}$ is decreasing and bounded, then $\lim_{n\to\infty} a_n = g.l.b.$.

(c) If $\lim_{n\to\infty} a_n = 0$, then $\{a_n\}$ converges to 0.

(d) If $\lim_{n\to\infty} a_n = 0$, then $\sum_n a_n$ converges.

(e) If $\sum_n a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

(f) If $\sum_n a_n$ diverges, then $\lim_{n\to\infty} a_n \neq 0$.

(g) If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_n a_n$ diverges.

(h) If $\lim_{n\to\infty} a_n \neq 0$, then $\{a_n\}$ diverges.

(i) If $\int_1^\infty f(x)dx = L$, where $0 < L < \infty$, then $\sum_n f(n) = L$.

(j) If $\sum_n b_n$ converges and $a_n > b_n$ for all $n \geq 1$, then $\sum_n a_n$ also converges.

(k) If $\sum_n b_n$ diverges and $a_n > b_n$ for all $n \geq 1$, then $\sum_n a_n$ also diverges.

(l) If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$ and $\sum_n b_n$ diverges, then $\sum_n a_n$ also diverges.

3. For each sequence below, find the l.u.b. and g.l.b., and determine if the sequence is monotonic.

$$\{\sin(n\pi)\}, \quad \left\{(-1)^{n+1}\frac{1}{5^n}\right\}, \quad \left\{\frac{n+1}{n}\right\}$$

4. Determine whether or not each sequence converges. If so, find the limit.

(a) $\left\{\frac{2n^2}{\sqrt{9n^4+1}}\right\}$

(b) $\left\{\left(1-\frac{1}{8n}\right)^n\right\}$

(c) $\left\{\frac{n!}{e^n}\right\}$

(d) $\left\{\left(\frac{n}{n+5}\right)^n\right\}$

5. Use series to write the repeating decimals (a) 0.31313131... and (b) $0.3\overline{27}$ as rational numbers.

6. **Find the sum** of each convergent series below, or explain why the series diverges.

(a) $\sum_{k=7}^\infty \frac{1}{(k-3)(k+1)}$

(b) $\sum_{k=0}^\infty (-1)^k$

(c) $\sum_{k=2}^\infty \frac{2^k+1}{3^{k+1}}$

(d) $\sum_{n=2}^\infty \frac{2^{2n-1}}{4\cdot 10^{n-1}}$

(e) $\sum_{n=2}^\infty \left(\frac{3}{n^2} - \frac{3}{(n+1)^2}\right)$

(f) $\sum_{n=1}^\infty \frac{3n+4}{n^3+3n^2+2n}$

7. Find a closed formula for the following sequences.

(a) $\{6, 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \frac{2}{81}, \ldots\}$

(b) $\{\frac{5}{1}, \frac{8}{2}, \frac{11}{6}, \frac{14}{24}, \frac{17}{120}, \ldots\}$

8. Determine if each series below converges or diverges. JUSTIFY YOUR ANSWER FULLY using any of the convergence tests from class.

(a) $\sum_{k=1}^\infty \frac{e^k}{4+e^{2k}}$

(b) $\sum_{k=1}^\infty \frac{5k^2+8}{7k^2+6k+1}$

(c) $\sum_{k=1}^{\infty} \frac{3^{2k}}{8^k - 3}$

(d) $\sum_{k=1}^{\infty} \frac{k+2}{\sqrt{k^5+4}}$

(e) $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^3}$

(f) $\sum_{k=1}^{\infty} k \tan\left(\frac{1}{k}\right)$

(g) $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \ldots$

(h) $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{n^2-1}}$

(i) $\sum_{k=1}^{\infty} \frac{\ln k}{k^4}$

(j) $\sum_{k=1}^{\infty} \frac{(2k)^k}{k!}$

(k) $\sum_{k=1}^{\infty} \left(\frac{k}{k+1}\right)^{2k^2}$

(l) $\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n-1)}{4^n 2^n n!}$

9. Suppose $r > 0$. Find the values of r, if any, for which $\sum_{k=1}^{\infty} \frac{r^k}{k^r}$ converges.

10. Determine whether the following alternating series converge absolutely, converge conditionally, or diverge. Justify your answers using the tests we discussed in class.

(a) $\sum_{k=2}^{\infty} (-1)^{k+1} \frac{3k}{\sqrt{k^3+4}}$

(b) $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k^4-1}$

(c) $\sum_{k=0}^{\infty} (-1)^k \frac{k}{5^k+2^k}$

(d) $\sum_{k=2}^{\infty} (-1)^k \frac{1}{k \ln k \sqrt{\ln \ln k}}$

(e) $\sum_{n=2}^{\infty} (-1)^n \frac{n}{n^2-1}$

(f) $\sum_{n=3}^{\infty} (-1)^n \frac{n-2}{n+2}$

(g) $\sum_{n=4}^{\infty} (-1)^n \frac{1}{n-3}$

(h) $\sum_{n=0}^{\infty} (-5)^{-n}$

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{\sqrt{n}}$

(j) $\sum_{n=2}^{\infty} (-1)^n \frac{1}{n^2-n}$

11. Find the radius and interval of convergence of the following power series:

(a) $\sum_{k=2}^{\infty} \left(\frac{k}{k-1}\right) \frac{(x+2)^k}{2^k}$

(b) $\sum_{k=1}^{\infty} \frac{k}{3^k(k^2+1)}(x-5)^k$

(c) $\sum_{k=1}^{\infty} \frac{5^k}{\sqrt{k}}(3-2x)^k$

(d) $\sum_{k=1}^{\infty} \frac{(-1)^k a^k}{k^2}(x-a)^k$, where $a \neq 0$

12. Determine if each series below converges or diverges. JUSTIFY YOUR ANSWER FULLY using any of the convergence tests from class.

(a) $\sum_{n=1}^{\infty} \frac{(2n+1)!}{n^{2n}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{2^n}\left(1+\frac{1}{2n}\right)^{n^2}$

(c) $\sum_{n=1}^{\infty} \frac{2^n+n^2}{3^n+n}$

(d) $\sum_{n=2}^{\infty} \frac{(1+(-1)^n)n^2}{(n-1)^3}$

(e) $\sum_{k=1}^{\infty} k^2 e^{-k^2}$

13. Find all p such that the infinite series $\sum_{n=3}^{\infty} \frac{1}{n^p \ln(n)}$ converges.

14. Consider the power series $f(x) = \sum_{n=0}^{\infty} \left(\frac{x^2+2}{6}\right)^n$.

(a) Find the interval of convergence for this series.

(b) For all values of x that lie in the interval of convergence, what is the sum of this series as a function of x?

15. Determine how many terms must be used to determine the sum of the entire series with an error of less than 0.01

(a) $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2+1}$

(b) $\sum_{n=0}^{\infty}\left(-\frac{1}{3}\right)^n$

(c) $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

16. For which value of p does the following series converge to 7?

$$\sum_{k=0}^{\infty} \frac{3}{2^{pk}}$$

Answers

2. Statements (b), (c), (e), (g), and (k) are true.

3. (a) l.u.b.=g.l.b.=0, monotonic

(b) l.u.b.=$\frac{1}{5}$, g.l.b.=$-\frac{1}{25}$, not monotonic

(c) l.u.b.=2, g.l.b.=1, monotonic

4. (a) converges to $\frac{2}{3}$, (b) converges to $e^{-1/8}$, (c) diverges, (d) converges to $\frac{1}{e^5}$

5. (a) $\frac{31}{99}$, (b) $\frac{108}{330}$

6. (a) ≈ 0.1899, (b) diverges, (c) $\frac{1}{2}$, (d) $\frac{1}{3}$, (e) $\frac{3}{4}$, (f) $\frac{5}{2}$

7. (a) $a_n = 2 \cdot 3^{2-n}$, $n \geq 1$ (b) $a_n = \frac{2+3n}{n!}$, $n \geq 1$

8. (a) converges by integral test, (b) diverges by divergence test, (c) diverges by direct comparison, (d) converges by direct or limit comparison, (e) converges by integral test, (f) diverges by divergence test, (g) converges by direct comparison, (h) converges by limit comparison, (i) converges by direct comparison, (j) diverges by ratio test, (k) converges by root test, (l) converges by ratio test

9. converges when $0 < r < 1$

10. (a) converges conditionally (limit comparison and alternating series test), (b) converges absolutely (limit comparison), (c) converges absolutely (ratio test), (d) converges conditionally (integral test and alternating series test), (e) conditionally convergent (direct comparison and alternating series test), (f) diverges (nth term test), (g) conditionally convergent (direct comparison and alternating series test), (h) absolutely convergent (geometric), (i) conditionally convergent (integral test and alternating series test), (j) absolutely convergent (limit comparison test)

11. (a) $R = 2$, $I.C. = (-4, 0)$, (b) $R = 3$, $I.C. = [2, 8)$ (c) $R = \frac{1}{10}$, $I.C. = \left(\frac{7}{5}, \frac{8}{5}\right]$, (d) $R = \frac{1}{|a|}$, $I.C. = [a - \frac{1}{|a|}, a + \frac{1}{|a|}]$,

12. (a) converges by ratio test, (b) converges by root test, (c) converges by limit comparison test, (d) diverges by direct comparison, (e) converges by the integral test

13. converges for $p > 1$, diverges for $p \leq 1$

14. $I.C. = (-2, 2)$, sum is $\frac{6}{4-x^2}$

15. (a) 9, (b) 4, (c) 100

16. $p = \log_2(7) - 2 \approx 0.8074$