

Math 1553 Exam 1, Fall 2026, Version B

Name	Key	GT ID		Section	
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Circle your instructor and lecture below. Be sure to circle the correct choice!

Callis (B, 8 AM)	Callis (C, 9 AM)	Fuertes (D, 10 AM)	Fuertes (E, 11 AM)
Barone (F, 11 AM)	Kassem (G, 12 PM)	Poudel (H, 1 PM)	Yildiz (I, 2 PM)
Chin (J, 2 PM)	Ning (L, 3 PM)	Tang (M, 4 PM)	Tang (N, 5 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means “reduced row echelon form.” The “zero vector” in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with “X” or “/” or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 2:00 PM on Tuesday next week, after the make-up exam.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

- (a) If the RREF of an augmented matrix has a pivot in every column, then the corresponding system of linear equations must be inconsistent.

☒ True

☐ False

- (b) If the RREF of an augmented matrix has no pivot in its rightmost column, then the corresponding system of linear equations must have a unique solution.

☐ True

☒ False

- (c) Suppose A is an $m \times n$ matrix and the columns of A span $\mathbf{R}^m$. Then the equation $Ax = b$ is consistent for every b in $\mathbf{R}^m$.

☒ True

☐ False

- (d) If the homogeneous matrix equation $Ax = 0$ has a free variable, then the system is consistent and it has infinitely many solutions.

☒ True

☐ False

- (e) Suppose A is a 3×2 matrix and $A \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

Then the matrix equation $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ must have infinitely many solutions.

☒ True

☐ False

2. Multiple choice and short answer. You do not need to show work or justify your answers. Parts (a) through (d) are unrelated. Parts (a) and (d) are worth 3 points each, while parts (b) and (c) are worth 2 points each.

- (a) Suppose that the solution set to a matrix equation $Ax = b$ has parametric form

$$x_1 = -2 + x_3, \quad x_2 = 3 - 2x_3, \quad x_3 = x_3 \text{ (} x_3 \text{ real)}, \quad x_4 = 3.$$

Which of the following statements **must** be true? Clearly mark all that apply.

- ☒ The vector $\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix}$ is a solution to the homogeneous matrix equation $Ax = 0$.
- ☒ The vector $\begin{pmatrix} -1 \\ 1 \\ 1 \\ 3 \end{pmatrix}$ is a solution to the inhomogeneous matrix equation $Ax = b$.
- ☐ The matrix A has 5 columns.

- (b) Suppose a 4×6 augmented matrix has 4 pivots and corresponds to a consistent system of linear equations. Fill in the bubble for exactly one choice.

(i) The solution set is:

- ☐ a point ☒ a line ☐ a plane ☐ $\mathbf{R}^3$ ☐ $\mathbf{R}^4$

(ii) The solution set lives in:

- ☐ $\mathbf{R}^2$ ☐ $\mathbf{R}^3$ ☐ $\mathbf{R}^4$ ☒ $\mathbf{R}^5$ ☐ $\mathbf{R}^6$

- (c) Consider the vector equation $x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Which of the following describes the solution set to the vector equation?

- ☒ a point in $\mathbf{R}^2$ ☐ a line in $\mathbf{R}^2$ ☐ all of $\mathbf{R}^2$
- ☐ a point in $\mathbf{R}^3$ ☐ a line in $\mathbf{R}^3$ ☐ a plane in $\mathbf{R}^3$ $\left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \end{array} \right]$

- (d) Which of the following matrices are in reduced row echelon form (RREF)? Clearly mark all that apply.

- ☐ $\left(\begin{array}{ccc|c} 1 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right)$
- ☒ $\left(\begin{array}{cccc|c} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$ ✓
- ☐ $\left(\begin{array}{ccc|c} 1 & 0 & \cancel{2} & 2 & 0 \\ 0 & 1 & \cancel{2} & -1 & 0 \\ 0 & 0 & \cancel{1} & 1 & 0 \end{array} \right)$

3. Short answer and multiple choice. Parts (a) through (c) are unrelated. There is no partial credit except on (a).

(a) (4 points) Consider the matrices $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. Determine whether the following statements are true or false, and clearly circle the appropriate answer.

i. The span of the columns of A is $\mathbf{R}^3$

☒ True ☐ False

ii. The span of the columns of B is $\mathbf{R}^2$

☐ True ☒ False

iii. For the vector $b = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}$, $Ax = b$ has exactly one solution.

☒ True ☐ False

iv. There is a vector d in $\mathbf{R}^3$ such that the matrix equation $Bx = d$ is inconsistent.

☒ True ☐ False

(b) (1 point) Compute the product $\begin{pmatrix} 2 & 1 & -1 \\ 0 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$. Select the correct answer below. $= \begin{pmatrix} 2-2-3 \\ 0-6+12 \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

☐ $\begin{pmatrix} -3 \\ 18 \end{pmatrix}$ ☐ $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ ☒ $\begin{pmatrix} -3 \\ 6 \end{pmatrix}$ ☐ $\begin{pmatrix} 1 \\ -6 \\ 12 \end{pmatrix}$ ☐ $\begin{pmatrix} -3 \\ 6 \\ 0 \end{pmatrix}$

☐ $\begin{pmatrix} 3 \\ 6 \\ 0 \end{pmatrix}$ ☐ $\begin{pmatrix} -3 \\ 0 \\ 2 \end{pmatrix}$ ☐ $\begin{pmatrix} 2 & -2 & -3 \\ 0 & -6 & 12 \end{pmatrix}$ ☐ none of these

(c) (3 points) Consider the following linear system of equations, where h is some real number:

$$2x + 3y = 1$$

$$4x + hy = 2$$

$$\left[\begin{array}{cc|c} 2 & 3 & 1 \\ 4 & h & 2 \end{array} \right] \sim \left[\begin{array}{cc|c} 2 & 3 & 1 \\ 0 & h-6 & 0 \end{array} \right]$$

Which **one** of the following statements is true?

☒ If $h = 6$, then the system has infinitely many solutions.

☐ If the system is consistent, it must have exactly one solution.

☐ There is exactly one value of h that makes the system consistent.

☐ If $h = 5$, then the system is inconsistent.

$$\left[\begin{array}{cc|c} 3 & -6 & 6 \\ -1 & 2 & h \end{array} \right] \sim \left[\begin{array}{cc|c} -1 & 2 & h \\ 0 & 0 & 6+3h \end{array} \right]$$

(d) (2 points) Find all values of h (if there are any) so that $\begin{pmatrix} 6 \\ h \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} -6 \\ 2 \end{pmatrix}$. Fill in the bubble for the correct answer below.

☐ $h = 0$ only ☒ $h = -2$ only ☐ $h = 2$ only ☐ $h = 6$ only

☐ $h = -6$ only ☐ $h = -12$ only ☐ all real h ☐ none of these

4. Short answer. You do not need to show your work on this page. Parts (a) through (d) are unrelated. Parts (a), (b), and (c) are worth 2 points each, and part (d) is worth 4 points.

(a) Consider the system of linear equations corresponding to the augmented matrix

$$\left(\begin{array}{ccc|c} 1 & -1 & 2 & 4 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right).$$

$x_2 + 3(z) = 1 \Rightarrow x_2 = -5$
 $x_1 + 5 + 2(z) = 4 \Rightarrow x_1 = -5$

Which one of the following vectors is a solution to the system? Choose exactly one.

☐ $\begin{pmatrix} -3 \\ -5 \\ 2 \end{pmatrix}$
☒ $\begin{pmatrix} -5 \\ -5 \\ 2 \end{pmatrix}$
☐ $\begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}$
☐ $\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$
☐ none of these

(b) Suppose the solution set to some matrix equation $Ax = b$ has parametric vector form $x = \begin{pmatrix} -3 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, where x_2 is any real number. Which of the following are solutions to $Ax = 0$? Fill in the bubble for all that apply.

☐ $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$
☒ $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
☒ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
☒ $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$

(c) In each case, determine if the span of the columns of the matrix is $\mathbf{R}^2$. Clearly mark your answer.

(i) $\begin{pmatrix} 2 & 0 & 6 \\ 1 & 0 & 3 \end{pmatrix}$ ☐ Yes ☒ No

(ii) $\begin{pmatrix} 0 & 3 \\ 1 & 2 \end{pmatrix}$ ☒ Yes ☐ No

(d) Let v_1, v_2, v_3 be vectors in $\mathbf{R}^3$, and let A be the matrix whose three columns are v_1, v_2, v_3 and suppose that A has three pivots. Which of the following statements must be true? Clearly mark all that apply.

☒ Every vector b in $\mathbf{R}^3$ is a linear combination of v_1, v_2 , and v_3 .

☒ $\text{Span}\{v_1, v_2, v_3\}$ is $\mathbf{R}^3$.

☒ v_3 is not a linear combination of v_1 and v_2 .

☒ the RREF of A is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

The rest of the exam is free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct. Parts (a) and (b) are unrelated.

5. (a) (5 pts) Solve the following linear system in the variables x_1 , x_2 , and x_3 :

$$\begin{aligned}x_1 + x_2 - x_3 &= 4 \\ -2x_1 - x_2 + 4x_3 &= 1.\end{aligned}$$

Write the solution set in parametric form.

$$\begin{aligned}\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ -2 & -1 & 4 & 1 \end{array} \right] &\sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & 1 & 2 & 9 \end{array} \right] \\ &\sim \left[\begin{array}{ccc|c} 1 & 0 & -3 & -5 \\ 0 & 1 & 2 & 9 \end{array} \right] \quad \swarrow x_3 \text{ free}\end{aligned}$$

$$\begin{cases} x_1 = -5 + 3x_3 \\ x_2 = 9 - 2x_3 \\ x_3 = x_3 \text{ (free)} \end{cases}$$

Check: $\begin{aligned} x_1 &= -5 & \text{eqn 1 } \checkmark \\ x_2 &= 9 & \text{eqn 2 } \checkmark \\ x_3 &= 0 \end{aligned}$

$$\begin{aligned}x_1 - 3x_3 &= -5 \\ x_2 + 2x_3 &= 9 \\ x_3 &= x_3 \text{ (free)}\end{aligned}$$

- (b) (5 points) Find all values of h and k so that the following system has infinitely many solutions:

$$2x + hy = 1$$

$$8x + 4y = k.$$

$$\left[\begin{array}{cc|c} 2 & h & 1 \\ 8 & 4 & k \end{array} \right]$$

$$\sim \left[\begin{array}{cc|c} 2 & h & 1 \\ 0 & 4-4h & k-4 \end{array} \right]$$

need $\begin{aligned} 4-4h &= 0 \\ \text{and } k-4 &= 0 \end{aligned} \Rightarrow \begin{aligned} h &= 1 \\ k &= 4 \end{aligned}$

$$\begin{aligned} h &= 1 \\ k &= 4 \end{aligned}$$

6. Free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct. Parts (a) and (b) are unrelated.

(a) (6 points) Consider the vector equation

$$x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 3 \\ 9 \end{pmatrix} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}.$$

i. Find the solutions to the vector equation, and write them in parametric **vector** form.

ii. Using your answer from part (i), draw the solution set for the vector equation below very carefully. You do not need to show your work for this part.

$$\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 3 & 9 & 6 \end{array} \right] \sim \left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 0 & 0 \end{array} \right]$$

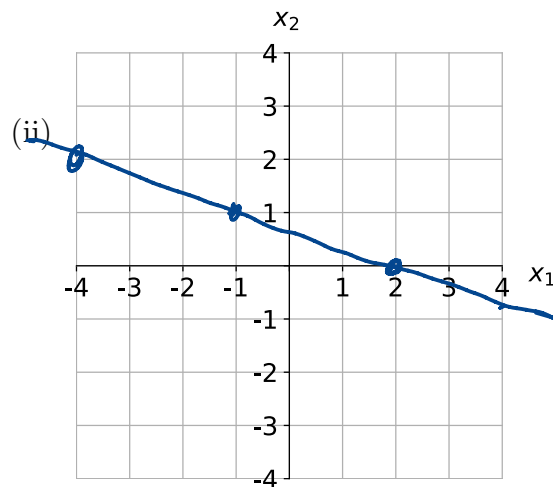
$\swarrow x_2$

$$x_1 = 2 - 3x_2$$

$$x_2 = x_2 \text{ (free)}$$

$$X = \begin{bmatrix} 2 - 3x_2 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$X = \text{(i)} \begin{bmatrix} 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$



(b) (4 points) Write a 3×3 matrix A with the property that $Ax = b$ is consistent if and only if b is in $\text{Span} \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \right\}$. You do not have to show work for this part.

many answer possible

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even if the answer is correct. Parts (a) and (b) are unrelated.

7. (a) (6 points) Find all values of h (if there are any) so that the vector $\begin{pmatrix} 3 \\ h \\ 4 \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$ $h =$ 5

$$\left(\begin{array}{cc|c} 1 & 1 & 3 \\ 2 & -1 & h \\ 1 & 4 & 4 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & -3 & h-6 \\ 0 & 3 & 1 \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 1 & 3 \\ 0 & 3 & -1 \\ 0 & 0 & h-5 \end{array} \right)$$

$$\text{need } h-5=0$$

$$\Rightarrow h=5$$

- (b) (4 points) Write an augmented matrix in reduced row echelon form, so that the set of solutions to the corresponding system of equations has parametric form given below. You do not need to show your work on this part.

$$x_1 = 3 - 2x_3, \quad x_2 = 4, \quad x_3 = x_3 \text{ (} x_3 \text{ real).}$$

$$\begin{aligned} x_1 + 2x_3 &= 3 \\ x_2 &= 4 \\ x_3 &= x_3 \text{ (free)} \end{aligned}$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 4 \end{array} \right)$$

This page is reserved **ONLY** for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.