Math 1553 Exam 2, Fall 2026, Version B

Name	Key	GT ID		Section	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Callis (B, 8 AM)	Callis (C, 9 AM)	Fuertes (D, 10 AM)	Fuertes (E, 11 AM)
Barone (F, 11 AM)	Kassem (G, 12 PM)	Poudel (H, 1 PM)	Yildiz (I, 2 PM)
Chin (J, 2 PM)	Ning (L, 3 PM)	Tang (M, 4 PM)	Tang (N, 5 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.
- For questions with an answer box, please put your answer in the box.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 2:00 PM on Tuesday next week, after the make-up exam.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) Suppose V is a 4-dimensional subspace of $\mathbf{R}^n$, and let v_1, v_2, v_3, and v_4 be four linearly independent vectors in V. Then $\text{Span}\{v_1, v_2, v_3, v_4\} = V$.

   ● True

   ○ False

   (b) If a homogeneous linear system consists of 4 equations in 8 variables, then its solution set is a subspace of $\mathbf{R}^4$.

   ○ True

   ● False

   (c) If $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors in $\mathbf{R}^3$, then the vector equation $x_1v_1 + x_2v_2 = 0$ must have a unique solution.

   ● True

   ○ False

   (d) Let V be the subspace consisting of all vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in $\mathbf{R}^3$ that satisfy the equation $x + 3y - 4z = 0$. Then $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \right\}$ is a basis for V.

   ○ True

   ● False

   ↖ not in V

   $2 + 3 - 8 \neq 0$

   (e) If $T : \mathbf{R}^n \to \mathbf{R}^m$ is a linear transformation and $n \leq m$, then T is one-to-one.

   ○ True

   ● False

2. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Let A be a 70×50 matrix with 30 pivots.

i. What is the dimension of the column space of A?

○ 20 ● 30 ○ 40 ○ 50 ○ 70

ii. What is the dimension of the null space of A?

● 20 ○ 30 ○ 40 ○ 50 ○ 70

iii. Complete the sentence: $\mathrm{Col}(A)$ is a subspace of...

○ $\mathbf{R}^{20}$ ○ $\mathbf{R}^{30}$ ○ $\mathbf{R}^{40}$ ○ $\mathbf{R}^{50}$ ● $\mathbf{R}^{70}$

iv. Complete the sentence: $\mathrm{Nul}(A)$ is a subspace of...

○ $\mathbf{R}^{20}$ ○ $\mathbf{R}^{30}$ ○ $\mathbf{R}^{40}$ ● $\mathbf{R}^{50}$ ○ $\mathbf{R}^{70}$

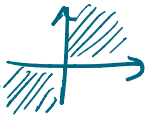
(b) (3 points) Suppose that v_1, v_2, and v_3 are linearly dependent vectors in $\mathbf{R}^n$. Which of the following must be true? Fill in the bubble for all that apply.

● The set $\{v_1, v_2, 5v_1 - 4v_2\}$ must be linearly dependent.

○ The set $\{v_1, v_2\}$ must be linearly dependent.

○ The equation $x_1v_1 + x_2v_2 = v_3$ must be consistent.

(c) (3 points) Let V be the set of vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbf{R}^2$ that satisfy $xy \geq 0$. Which of the subspace properties does V satisfy? Fill in the bubble for all that apply.



● V contains the zero vector.

○ V closed under addition. In other words, if u and v are vectors in V, then $u + v$ must be in V.

● V closed under scalar multiplication. In other words, if u is a vector in V and c is a scalar, then cu must be in V.

(d) (2 points) Suppose v_1, v_2, and v_3 are vectors in $\mathbf{R}^n$, and let A be the matrix whose columns are v_1, v_2, and v_3. Which **one** of the following statements guarantees that $\{v_1, v_2, v_3\}$ is linearly independent?

○ No column of A is a scalar multiple of any other column of A.

● A has three rows and every row of the RREF of A has a pivot.

○ The zero vector is a solution to $Ax = 0$.

○ For every vector b in $\mathbf{R}^n$, the equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ has at least one solution.

3. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (3 points) Which of the following sets of vectors are linearly independent? Fill in the bubble for all that apply.

○ $\left\{\begin{pmatrix}2\\1\\3\end{pmatrix}, \begin{pmatrix}1\\3\\2\end{pmatrix}, \begin{pmatrix}7\\11\\12\end{pmatrix}\right\}$

● $\left\{\begin{pmatrix}1\\-3\\2\end{pmatrix}, \begin{pmatrix}1\\2\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\end{pmatrix}\right\}$

○ $\left\{\begin{pmatrix}2\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\0\end{pmatrix}, \begin{pmatrix}3\\1\\1\end{pmatrix}\right\}$

(b) (3 points) Which of the following are subspaces of $\mathbf{R}^3$? Fill in the bubble for all that apply.

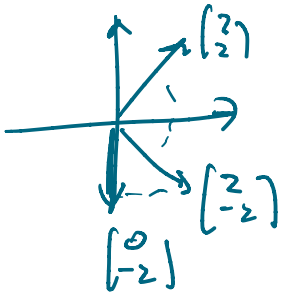
● The set of all vectors $x = \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$ in $\mathbf{R}^3$ satisfying $Ax = 0$ where $A = \begin{pmatrix}1&1&2\\2&2&4\end{pmatrix}$.

● The range of the linear transformation $T\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}1&0&2\\1&0&2\\1&0&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$.

● Span $\left\{\begin{pmatrix}1\\-3\\0\end{pmatrix}, \begin{pmatrix}2\\1\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}\right\}$

(c) (2 pts) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation that rotates each vector $\begin{pmatrix}x\\y\end{pmatrix}$ clockwise by 90°, then projects the vector to the y-axis.
Find $T\begin{pmatrix}2\\2\end{pmatrix}$.

○ $\begin{pmatrix}2\\0\end{pmatrix}$ ○ $\begin{pmatrix}-2\\0\end{pmatrix}$ ○ $\begin{pmatrix}0\\2\end{pmatrix}$ ● $\begin{pmatrix}0\\-2\end{pmatrix}$ ○ $\begin{pmatrix}-2\\-2\end{pmatrix}$



(d) (2 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be a linear transformation satisfying $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-1\\5\end{pmatrix}$ and $T\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}-2\\-1\end{pmatrix}$. Find $T\begin{pmatrix}-2\\-1\end{pmatrix}$.

○ $\begin{pmatrix}4\\-11\end{pmatrix}$ ○ $\begin{pmatrix}0\\1\end{pmatrix}$ ○ $\begin{pmatrix}-3\\4\end{pmatrix}$ ● $\begin{pmatrix}4\\-9\end{pmatrix}$ ○ none of these

4. On this page, you do not need to show work. Only your answers are graded. Parts (a) through (c) are unrelated.

(a) (5 points) Let $T : \mathbf{R}^n \to \mathbf{R}^m$ be a linear transformation with standard matrix A, so $T(x) = Ax$. Which of the following statements guarantee that T is one-to-one? Fill in the bubble for all that apply.

- ○ $n = m = 2$ and T projects vectors to the y-axis.
- ● For each y in $\mathbf{R}^m$, there is at most one x in $\mathbf{R}^n$ so that $T(x) = y$.
- ● Rank$(A) = n$.
- ● The equation $Ax = b$ has at most one solution for every b in $\mathbf{R}^m$.
- ● $n = m$ and T is onto.

(b) (2 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that rotates vectors **clockwise** by the angle θ. Which of the following is the standard matrix A for T? Fill in the bubble for the correct answer.

- ○ $A = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$
- ○ $A = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix}$
- ● $A = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}$
- ○ $A = \begin{pmatrix} \sin(\theta) & -\cos(\theta) \\ \cos(\theta) & \sin(\theta) \end{pmatrix}$

(c) (3 points) Suppose A is a matrix with 3 columns v_1, v_2, and v_3 (in that order), and suppose that the vector $r = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ is in the null space of A.

In the **left** box below, write another nonzero vector x (with $x \neq r$) in Nul A.
In the **right** box below, write a linear dependence relation for v_1, v_2, and v_3, that is, write a linear combination of v_1, v_2, v_3 which equals the zero vector.

$\begin{bmatrix} 2 \\ 4 \\ -6 \end{bmatrix}$	$v_1 + 2v_2 - 3v_3 = 0$

Free response. Show your work for part (b) only! A correct answer without sufficient work may receive little or no credit.

5. (10 points) For this page of the exam, consider the matrix A and its reduced row echelon form:

$$A = \begin{pmatrix} 1 & 3 & -2 & 1 \\ 2 & 6 & -4 & 1 \\ 5 & 15 & -10 & 1 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 3 & -2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Let T be the matrix transformation $T(x) = Ax$.

(a) Write a basis for $\mathrm{Col}(A)$. There is no work required on this part.

(b) Find a basis for $\mathrm{Nul}(A)$. Show your work.

from RREF of A

$x_1 + 3x_2 - 2x_3 = 0$ $\quad\Rightarrow\quad$ $x_1 = -3x_2 + 2x_3$

$x_2 = x_2$ (free) $\quad\Rightarrow\quad$ $x_2 = x_2$ (free)

$x_3 = x_3$ (free) $\quad\Rightarrow\quad$ $x_3 = x_3$ (free)

$x_4 = 0$ $\quad\Rightarrow\quad$ $x_4 = 0$

So $X = x_2 \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

Col(A) basis

$\left\{ \begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

Nul(A) basis

$\left\{ \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \right\}$

(c) Write one vector x so that $T(x) = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. There is no work required on this part.

↖ last column of A

$x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

(d) Let $u = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 2 \end{pmatrix}$. Find a vector v which is **different** from u (so $u \neq v$) satisfying $T(u) = T(v)$. There is no work required on this part.

$u + \begin{pmatrix} -3 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ works

$v = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 2 \end{pmatrix}$

Free response. Show your work unless otherwise indicated! A correct answer without appropriate work will receive little or no credit.

6. (10 points) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation $T(x_1, x_2, x_3) = (x_1 - 3x_3, -x_1 + 2x_2)$, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that reflects vectors across the x-axis.

   (a) Write the standard matrix A for T. Show your work.

$$A = \begin{pmatrix} 1 & 0 & -3 \\ -1 & 2 & 0 \end{pmatrix}$$

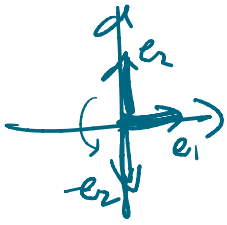
$A = [T(e_1)\ T(e_2)\ T(e_3)]$

$= \begin{bmatrix} 1 & 0 & -3 \\ -1 & 2 & 0 \end{bmatrix}$

   (b) Write the standard matrix B for U. Show your work.

$$B = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$B = [T(e_1)\ T(e_2)]$



   (c) Which composition makes sense: $T \circ U$ or $U \circ T$? Fill in the correct bubble below. You do not need to show your work on this part.

$2\times 2 \quad 3\times 2$

AB or BA ✓

   ◯ $T \circ U$ ◯ $U \circ T$

   (d) Compute the standard matrix C for the composition you selected in (c). Show your work.

$$C = \begin{pmatrix} 1 & 0 & -3 \\ 1 & -2 & 0 \end{pmatrix}$$

$BA = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & -3 \\ -1 & 2 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & -3 \\ 1 & -2 & 0 \end{pmatrix}$

7. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work will receive little or no credit. Parts (a) through (c) are unrelated.

(a) (3 points) Let V be the subspace of $\mathbf{R}^4$ consisting of all vectors of the form

$$\begin{pmatrix} a+b \\ b \\ a-c \\ a+c \end{pmatrix},$$

where a, b, and c are real. Find a basis for V. You do not need to show work on this part.

$$\begin{bmatrix} a+b \\ b \\ a-c \\ a+c \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + c\begin{bmatrix} 0 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} \right\}$$

(b) (4 pts) Find the values of x and y so that $A^2 = A$ for the matrix $A = \begin{pmatrix} 2 & x \\ 5 & y \end{pmatrix}$. Enter your answer below:

$x = \underline{-2/5}$ $\qquad$ $y = \underline{-1}$.

need $A^2 = A$

$$A^2 = \begin{pmatrix} 2 & x \\ 5 & y \end{pmatrix}\begin{pmatrix} 2 & x \\ 5 & y \end{pmatrix} = \begin{pmatrix} 4+5x & 2x+xy \\ 10+5y & 2x+y^2 \end{pmatrix} = \begin{pmatrix} 2 & x \\ 5 & y \end{pmatrix}$$

So $\quad 4+5x = 2 \quad \Rightarrow \quad 5x = -2 \quad x = -2/5$

$\qquad 10+5y = 5 \qquad\quad 5y = -5 \quad y = -1$

(c) (3 pts) Let $A = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 4 & 0 \\ 3 & 2 & -1 \end{pmatrix}$. Find all values of c (if there are any) so that $\begin{pmatrix} 4 \\ -1 \\ c \end{pmatrix}$ is in the null space of A. Enter your answer in the box below.

x in $\mathrm{Nul}\,A$ if $Ax = 0$ so

$c = 10$

$$Ax = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 4 & 0 \\ 3 & 2 & -1 \end{pmatrix}\begin{pmatrix} 4 \\ -1 \\ c \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{cases} 0 = 0 \\ 4-4 = 0 \\ 12-2-c = 0 \end{cases}$$

$\Rightarrow \quad c = 10$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.