Math 1553 Exam 1, Summer 2026, Ver. A

Name	Key	GT ID		Section	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Barone (E, 2:00 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 2:00 PM on Tuesday next week, after the make-up exam.

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Su'26

1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

(a) If an augmented matrix in RREF has a pivot in every row, then its corresponding system of linear equations must be consistent.

○ True

● False

(b) If the bottom row of an augmented matrix corresponding to a system of two linear equations in the three variables x, y, z in row echelon form is $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 3 \end{array}\right)$, then the corresponding system of linear equations must have infinitely many solutions.

○ True

● False

(c) If the zero vector is a **not** solution to a matrix equation, then the matrix equation must be inhomogeneous.

● True

○ False

(d) Suppose u, v, and w are vectors in $\mathbf{R}^3$ and $\mathrm{Span}\{u, v, w\} = \mathbf{R}^3$. If b is any vector in $\mathbf{R}^3$ then $x_1u + x_2v + x_3w = b$ must have exactly one solution.

● True

○ False

(e) Suppose A is a 2×3 matrix and $A \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Then the matrix equation $Ax = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ must have infinitely many solutions.

● True

○ False

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2. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (3 points) Which of the following matrices are in RREF? Fill in the bubble for all that apply.

● $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right)$

● $\left(\begin{array}{cccc|c} 0 & 1 & 3 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

● $\left(\begin{array}{cccc|c} 1 & -13 & -5 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

2 pivot 2 free

4 1

(b) (2 points) Suppose an augmented matrix with 3 rows and 5 columns (including its right-most column) has 2 pivots and corresponds to a consistent system of linear equations. Fill in the bubble for exactly one choice.

(i) The solution set is:

○ a point ○ a line ● a plane ○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$

(ii) The solution set lives in:

○ $\mathbf{R}$ ○ $\mathbf{R}^2$ ○ $\mathbf{R}^3$ ● $\mathbf{R}^4$ ○ $\mathbf{R}^5$

(c) (2 points) Consider the vector equation $x_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Which of the following describes the solution set to the vector equation?

○ a point in $\mathbf{R}^2$ ○ a line in $\mathbf{R}^2$ ○ all of $\mathbf{R}^2$

○ a point in $\mathbf{R}^3$ ● a line in $\mathbf{R}^3$ ○ a plane in $\mathbf{R}^3$

(d) (3 points) Suppose v_1, v_2, and v_3 are vectors in $\mathbf{R}^2$. Which of the following statements are true? Fill in the bubble for all that apply.

○ If v_1 is not the zero vector, then Span $\{v_1\}$ consists of exactly one vector.

● The vector equation $x_1v_1 + x_2v_2 + x_3v_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ must have infinitely many solutions.

○ If the solution set to $x_1v_1 + x_2v_2 + x_3v_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is inconsistent, then the solution set to the homogeneous equation $x_1v_1 + x_2v_2 + x_3v_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ must also be inconsistent.

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3. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (3 points) Suppose that the solution set to a matrix equation $Ax = b$ has parametric form

$$x_1 = 2 + x_3, \qquad x_2 = 3 - x_3, \qquad x_3 = x_3 \ (x_3 \text{ real}), \qquad x_4 = 5.$$

Which of the following statements **must** be true? Fill in the bubble for all that apply.

● The vector $\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}$ is a solution to the homogeneous matrix equation $Ax = 0$.

○ b is a vector in $\mathbf{R}^4$.

● The matrix A has 4 columns.

(b) (2 points) Compute the product $\begin{pmatrix} 1 & -4 & -2 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} = \begin{bmatrix} 2+4 \\ 6-1 \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$

Fill in the bubble for the correct answer below.

○ $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ ○ $\begin{pmatrix} -2 \\ 7 \end{pmatrix}$ ○ $\begin{pmatrix} 6 \\ 7 \end{pmatrix}$ ● $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$ ○ $\begin{pmatrix} 6 \\ 5 \\ 0 \end{pmatrix}$

○ $\begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} -2 \\ 7 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 2 & 4 & 0 \\ 6 & -1 & 0 \end{pmatrix}$ ○ none of these

(c) (3 points) Consider the following linear system of equations, where h is some real number:

$$\begin{aligned} 3x + 2y &= 1 \\ 6x + hy &= 2 \end{aligned}$$

$$\left[\begin{array}{cc|c} 3 & 2 & 1 \\ 6 & h & 2 \end{array}\right] \sim \left[\begin{array}{cc|c} 3 & 2 & 1 \\ 0 & h-4 & 0 \end{array}\right]$$

Which **one** of the following statements is true?

● If $h = 4$, then the system has infinitely many solutions.

○ If the system is consistent, it must have exactly one solution.

○ There is exactly one value of h that makes the system consistent.

○ If $h = 5$, then the system is inconsistent.

(d) (2 points) Find all values of h (if there are any) so that $\begin{pmatrix} -4 \\ h \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 2 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Fill in the bubble for the correct answer below.

○ $h = 0$ only ○ $h = 2$ only ○ $h = 4$ only ○ $h = -4$ only

○ $h = -8$ only ● $h = 8$ only ○ all real h ○ none of these

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4. On this page, you do not need to show work. Only your answers are graded. Parts (a)-(c) are unrelated.

(a) (3 points) Suppose A is an $m \times n$ matrix and b is in $\mathbf{R}^m$. Which of the following conditions **guarantee** that $Ax = b$ is consistent? Clearly fill in the bubble for all that apply.

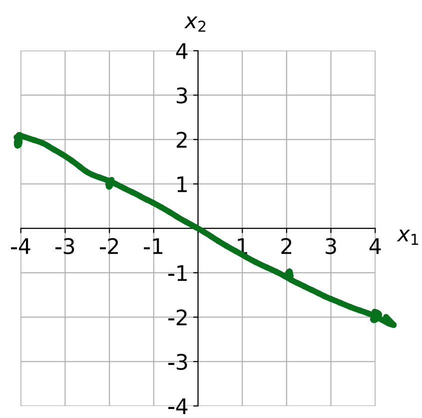
- ● b is in the span of the columns of A.
- ○ A has a pivot in every column.
- ○ The augmented matrix $(\, A \mid b \,)$ has a pivot in every row.

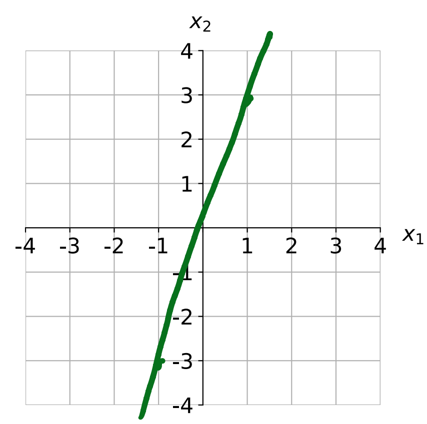
(b) (3 points) In the space provided, write three **different** vectors v_1, v_2, v_3 in $\mathbf{R}^3$ so that $\mathrm{Span}\{v_1, v_2, v_3\}$ is a line.

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad v_3 = \begin{pmatrix} 3 \\ 3 \\ 3 \end{pmatrix}$$

(c) (4 points) For some 2×2 matrix A, the vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is in the span of the columns of A, and the vector $x = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ is a solution to $Ax = 0$.

On the **left** graph below, carefully draw the span of the columns of A.
On the **right** graph below, carefully draw the solution set for $Ax = 0$.





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The rest of the exam is free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct. Parts (a) and (b) are unrelated.

5. (a) (5 pts) Solve the following linear system in the variables x_1, x_2, and x_3:

$$x_1 + x_2 - x_3 = 5$$

$$-x_1 + x_2 + 3x_3 = 1.$$

Write the solution set in parametric form.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ -1 & 1 & 3 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -1 & 5 \\ 0 & 2 & 2 & 6 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & -2 & 2 \\ 0 & 1 & 1 & 3 \end{array}\right]$$

$$\boxed{\begin{array}{l} x_1 = 2 + 2x_3 \\ x_2 = 3 - x_3 \\ x_3 = x_3 \text{ (free)} \end{array}}$$

(b) (5 points) Find all values of h and k so that the following system has exactly one solution:

$$2x - hy = 5$$

$$6x + 9y = k.$$

$$\boxed{\begin{array}{l} h \neq -3 \\ k \text{ anything} \end{array}}$$

$$\left[\begin{array}{cc|c} 2 & -h & 5 \\ 6 & 9 & k \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & -h & 5 \\ 0 & 9+3h & k-15 \end{array}\right]$$

need $9+3h \neq 0$, k can be anything

$h \neq -3$

Su'26

Free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct. Parts (a) and (b) are unrelated.

(a) (6 points) Consider the vector equation

$$x_1 \begin{pmatrix} 1 \\ 2 \end{pmatrix} + x_2 \begin{pmatrix} -3 \\ -6 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix}.$$

i. Find the set of solutions to the vector equation, and write it in vector parametric form.

ii. Using your answer from part (i), draw the solution set for the vector equation below very carefully. You do not need to show your work for this part.

free

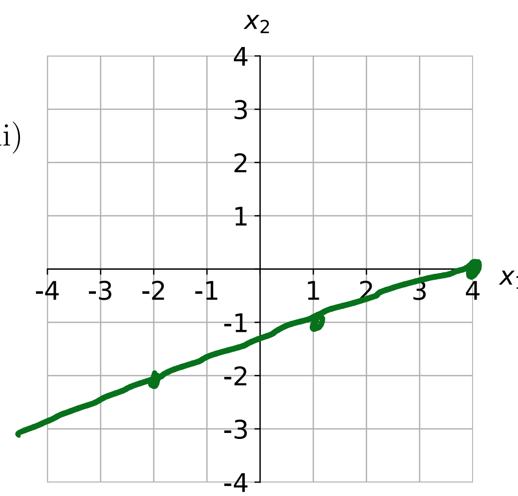
$$\left[\begin{array}{cc|c} 1 & -3 & 4 \\ 2 & -6 & 8 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -3 & 4 \\ 0 & 0 & 0 \end{array}\right]$$

$$x_1 = 4 + 3x_2$$
$$x_2 = x_2 \text{ (free)}$$

$$x = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

(i) $x = \begin{bmatrix} 4 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(ii)



(b) (4 points) Write a 3×3 matrix A with the property that $Ax = b$ is consistent if and only if b is in Span $\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\}$. You do not have to show work for this part.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

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Free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even if the answer is correct. Parts (a) and (b) are unrelated.

6. (a) (6 points) Find all values of h (if there are any) so that the vector $\begin{pmatrix}4\\h\\2\end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix}1\\1\\2\end{pmatrix}$ and $\begin{pmatrix}2\\-3\\-1\end{pmatrix}$

$h = \boxed{-2}$

$$\left[\begin{array}{cc|c}1 & 2 & 4\\1 & -3 & h\\2 & -1 & 2\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 2 & 4\\0 & -5 & h-4\\0 & -5 & -6\end{array}\right] \sim \left[\begin{array}{cc|c}1 & 2 & 4\\0 & -5 & h-4\\0 & 0 & -(h-4)-6\end{array}\right]$$

need $-h+4-6=0$ so $\boxed{h=-2}$

(b) (4 points) Write an augmented matrix in reduced row echelon form, so that the set of solutions to the corresponding system of equations has parametric form given below. You do not need to show your work on this part.

$$x_1 = 1 + 2x_3, \qquad x_2 = 3, \qquad x_3 = x_3 \ (x_3 \text{ real}).$$

$$\left[\begin{array}{ccc|c}1 & 0 & -2 & 1\\0 & 1 & 0 & 3\end{array}\right]$$

$$\boxed{\left[\begin{array}{ccc|c}1 & 0 & -2 & 1\\0 & 1 & 0 & 3\end{array}\right]}$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.