Math 1553 Exam 2, Summer 2026, Ver. A

Name	Key	GT ID		Section	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Nurhan (D, 12:30 PM) Barone (E, 2:00 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- Simplify your answers.
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 2:00 PM on Tuesday next week, after the make-up exam.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) Suppose v_1, v_2, and v_3 are vectors in $\mathbf{R}^3$ and that there is exactly one solution to the equation

   $$x_1v_1 + x_2v_2 + x_3v_3 = 0.$$

   Then the equation $x_1v_1 + x_2v_2 = 0$ must also have exactly one solution.

   - True
   - False

   (b) Let V be the subspace consisting of all vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ in $\mathbf{R}^3$ that satisfy the equation $2x + y - 2z = 0$. Then $\left\{ \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 4 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ is a basis for V.

   - True
   - False

   (c) Suppose $T : \mathbf{R}^n \to \mathbf{R}^n$ is a linear transformation and T is not onto, then T cannot be one-to-one.

   - True
   - False

   (d) If A is a 5×9 matrix, then $\dim(\text{Nul } A) > \dim(\text{Col } A)$.

   - True
   - False

   could have 5 pivots.

   9

   5

   at most 5 pivots

   4 or more free vars.

   (e) Suppose A is a 3×5 matrix and B is a 5×3 matrix, and let T be the linear transformation given by $T(x) = BAx$. Then the domain of T is $\mathbf{R}^3$ and the codomain of T is $\mathbf{R}^3$.

   - True
   - False

   5x3 3x5

2. Multiple choice. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (3 points)Suppose that v_1, v_2, and v_3 are linearly independent vectors in $\mathbf{R}^n$. Which of the following must be true? Fill in the bubble for all that apply.

● The vector equation $x_1v_1 + x_2v_2 + x_3v_3 = 0$ is consistent and has exactly one solution.

○ The set $\{v_1, v_2, v_1 - 2v_2\}$ must be linearly dependent.

● The equation $x_1v_1 + x_2v_2 = v_3$ must be inconsistent.

(b) (2 points) Suppose A is a 40×25 matrix whose RREF has 10 pivots. Which one of the following describes the null space of A?

● Nul(A) is a 15-dimensional subspace of $\mathbf{R}^{25}$.

○ Nul(A) is a 30-dimensional subspace of $\mathbf{R}^{25}$.

○ Nul(A) is an 15-dimensional subspace of $\mathbf{R}^{40}$.

○ Nul(A) is an 30-dimensional subspace of $\mathbf{R}^{40}$.

10 pivot
15 free

25

40

(c) (3 points) Let V be the set of vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbf{R}^2$ that satisfy $xy = 0$. Which of the subspace properties does V satisfy? Fill in the bubble for all that apply.

● V contains the zero vector.

○ V is closed under addition. In other words, if u and v are vectors in V, then $u + v$ must be in V.

● V is closed under scalar multiplication. In other words, if u is a vector in V and c is a scalar, then cu must be in V.

(d) (2 points) Suppose A is a 3×5 matrix and B is a 5×4 matrix. Which **one** of the following statements must be true?

● If x is a vector in Nul(B), then $ABx = 0$.

○ $\dim \mathrm{Col}(A) + \dim \mathrm{Nul}(A) = 3$.

○ The set of all vectors b which are a linear combination of the columns of B is a subspace of $\mathbf{R}^4$.

○ If the homogeneous equation $Bx = 0$ has only the trivial solution, then the equation $ABx = 0$ has only the trivial solution.

3. Multiple choice. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Suppose that $T : \mathbf{R}^n \to \mathbf{R}^m$ is a transformation. Which **one** of the following conditions guarantees that T is one-to-one?

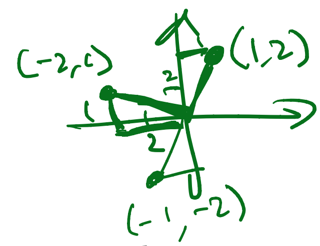
○ For each x in $\mathbf{R}^n$, there is at least one y in $\mathbf{R}^m$ so that $T(x) = y$.

○ For each x in $\mathbf{R}^n$, there is at most one y in $\mathbf{R}^m$ so that $T(x) = y$.

○ For each y in $\mathbf{R}^m$, there is at least one x in $\mathbf{R}^n$ so that $T(x) = y$.

● For each y in $\mathbf{R}^m$, there is at most one x in $\mathbf{R}^n$ so that $T(x) = y$.

○ For some y in $\mathbf{R}^m$, the equation $T(x) = y$ is inconsistent.



(b) (2 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that rotates vectors by 90° counterclockwise. Find the vector v so that $T(v) = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$. Fill in the bubble for the correct answer below.

● $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ ○ $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$

(c) (2 points) Which **one** of the following linear transformations is onto? Clearly fill in the bubble for your answer.

● $T : \mathbf{R}^3 \to \mathbf{R}^3$ that reflects vectors across the xy-plane.

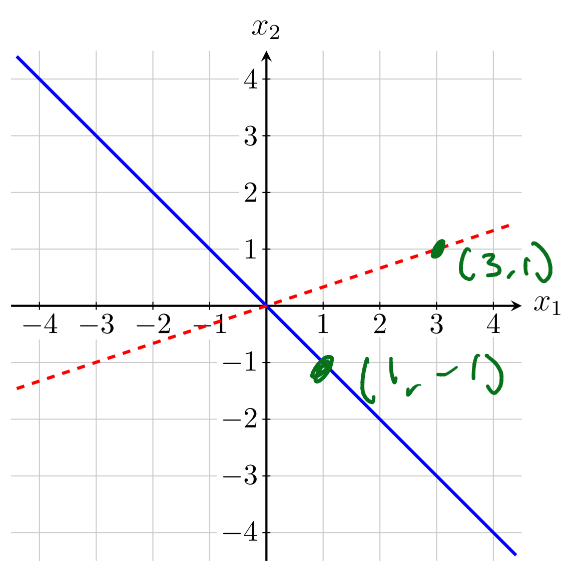
○ $T : \mathbf{R}^2 \to \mathbf{R}^3$ given by $T(x, y) = (x + y, 0, x - y)$.

○ $T : \mathbf{R}^4 \to \mathbf{R}^2$ given by $T(x, y, z, w) = (x - y, 2y - 2x)$.

○ $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by $T(x, y, z) = (x, y, 0)$.

(d) (4 points) Write a single matrix A whose column space is the **solid** line below and whose null space is the **dashed** line below.

$$\begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix}$$



$$\begin{bmatrix} 1 & -3 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

4. Short answer. You do not need to show your work. Parts (a)-(d) are unrelated.

(a) (2 points) Suppose A is an $m \times n$ matrix with $m = n + 1$, and let T be the matrix transformation $T(x) = Ax$. Which **one** of the following statements is correct?

- T must be one-to-one, but T cannot be onto.
- T cannot be one-to-one, but T must be onto.
- T cannot be one-to-one, but we need more information to determine whether T is onto.
- T cannot be onto, but we need more information to determine whether T is one-to-one.

$$5\overset{4}{\left(\,||||\,\right)} \text{ tall}$$

(b) (3 points) Which of the following transformations are linear? Clearly fill in the bubble for all that apply.

- $T : \mathbf{R}^2 \to \mathbf{R}^3$ given by $T(x, y) = (2x, 0, 0)$.
- $T : \mathbf{R}^3 \to \mathbf{R}^2$ given by $T(x, y, z) = (y, -y)$.
- $T : \mathbf{R}^2 \to \mathbf{R}^2$ given by $T(x, y) = (\sqrt{2}x - \pi y, x)$.

(c) (2 points) Consider the matrix below and its reduced row echelon form.

$$A = \begin{pmatrix} 2 & 4 & 1 & 8 \\ 2 & 4 & 1 & 8 \\ -1 & -2 & 0 & -3 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Write a basis for the column space of A in the box provided below.

$$\left\{ \begin{bmatrix} 2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

(d) (3 points) Suppose A is a matrix with 3 columns v_1, v_2, and v_3 (in that order), and suppose that the vector $r = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ is in the null space of A.

In the **left** box below, write another nonzero vector x (with $x \neq r$) in Nul A.

In the **right** box below, write a linear dependence relation for v_1, v_2, and v_3, that is, write a linear combination of v_1, v_2, v_3 which equals the zero vector.

$$\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$$

$$v_1 - v_2 + 2v_3 = 0$$

5. Short answer and free response. Parts (a) through (c) are unrelated. You do not need to show your work on (a) or (b), but show your work on part (c) to receive credit.

(a) (2 points) Suppose a linear transformation T satisfies

$$T\begin{pmatrix}2\\1\end{pmatrix} = \begin{pmatrix}-1\\3\end{pmatrix} \quad \text{and} \quad T\begin{pmatrix}0\\2\end{pmatrix} = \begin{pmatrix}2\\4\end{pmatrix}.$$

Compute $T(v)$ for the vector $v = 2\begin{pmatrix}2\\1\end{pmatrix} - 3\begin{pmatrix}0\\2\end{pmatrix}$. $\begin{bmatrix}3\end{bmatrix}$ $\begin{pmatrix}4\end{pmatrix}$ $\begin{bmatrix}6 & 12\end{bmatrix}$

- ○ $\begin{pmatrix}4\\-4\end{pmatrix}$
- ○ $\begin{pmatrix}1\\7\end{pmatrix}$
- ○ $\begin{pmatrix}-2 & -6\\6 & -12\end{pmatrix}$
- ○ $\begin{pmatrix}-8\\18\end{pmatrix}$
- ○ $\begin{pmatrix}2\\-1\end{pmatrix}$
- ● $\begin{pmatrix}-8\\-6\end{pmatrix}$
- ○ $\begin{pmatrix}-4\\-6\end{pmatrix}$
- ○ $\begin{pmatrix}-5\\-18\end{pmatrix}$
- ○ $\begin{pmatrix}2 & 0\\1 & 2\end{pmatrix}$
- ○ none of these

(b) (4 points) Write the standard matrix A for a linear transformation T satisfying all of the following conditions.

i. The range of T is the line y-axis in $\mathbf{R}^2$.

ii. The set of solutions to the equation $T(x) = 0$ is a plane.

$$A = \begin{pmatrix}0 & 0 & 0\\1 & 1 & 1\end{pmatrix}$$

(c) (4 pts) Find the values of x and y so that $A^2 = A$ for the matrix $A = \begin{pmatrix}x & y\\4 & -2\end{pmatrix}$. Enter your answer below:

$x =$ 3 $\qquad y =$ $-3/2$

$$\begin{bmatrix}x & y\\4 & -2\end{bmatrix}\begin{bmatrix}x & y\\4 & -2\end{bmatrix} = \begin{bmatrix}x^2+4y & xy-2y\\4x-8 & 4y+4\end{bmatrix}$$

$4x-8 = 4 \Rightarrow 4x = 12$

$\Rightarrow x = 3$

$4y+4 = -2 \Rightarrow 4y = -6$

$\Rightarrow y = -6/4 = -3/2$

Short answer and free response. You do not need to show your work on (a) - (c), but show your work on part (d) to receive credit.

6. (10 points) Let $T : \mathbf{R}^2 \to \mathbf{R}^3$ be the transformation $T(x, y) = (3x - 2y, y, -x + 5y)$, and let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the transformation that rotates vectors $90°$ counterclockwise.

(a) Find the standard matrix A for T and write it in the space below.

$$A = \begin{pmatrix} 3 & -2 \\ 0 & 1 \\ -1 & 5 \end{pmatrix}$$

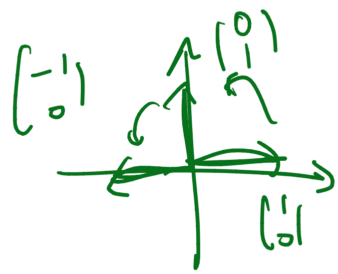
$T(1,0) = (3,0,-1) \leftarrow$ first col

$T(0,1) = (-2,1,5) \leftarrow$ second col

3×2

(b) Write the standard matrix B for U. Evaluate any trigonometric functions you write. Do not leave your answer in terms of sine and cosine.

$$B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$



2×2

(c) Which composition makes sense: $T \circ U$ or $U \circ T$? Fill in the correct bubble below. You do not need to show your work on this part.

● $T \circ U$ ○ $U \circ T$

AB so $T \circ U$

(d) Compute the standard matrix C for the composition you selected in (c). Put your answer in the space provided below. Show your work for this part.

$$C = \begin{pmatrix} -2 & -3 \\ 1 & 0 \\ 5 & 1 \end{pmatrix}$$

$$AB = \begin{pmatrix} 3 & -2 \\ 0 & 1 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{bmatrix} -2 & -3 \\ 1 & 0 \\ 5 & 1 \end{bmatrix}$$

7. Free response. Show your work for parts (b) and (c). A correct answer without sufficient work will receive little or no credit. Parts (a) through (c) are unrelated.

(a) (3 points) Let V be the subspace of $\mathbf{R}^4$ consisting of all vectors of the form

$$\begin{pmatrix} -4a+3b \\ b \\ a+b \\ b \end{pmatrix},$$

where a and b are real. Find a basis for V. You do not need to show work on this problem.

x in V if

$$X = a\begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \end{bmatrix} + b\begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\boxed{\left\{ \begin{bmatrix} -4 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}}$$

(b) (3 points) Let T be the linear transformation $T(x) = Ax$, where A is the matrix $A = \begin{pmatrix} 4 & -1 & 3 \\ 0 & 1 & 2 \end{pmatrix}$. Find $T\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$. Enter your answer in the space below, and show your work.

$$\begin{bmatrix} 4 & -1 & 3 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4-2-3 \\ 0+2-2 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} -1 \\ 0 \end{bmatrix}}$$

(c) (4 points) Find the inverse of the matrix A. Show your work.

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & 1 & 4 \end{pmatrix}$$

$$[A|I] = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array}\right)$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 1 & 3 & -1 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 2 & 3 & -2 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{array}\right]$$

$$\boxed{\begin{bmatrix} 2 & 1 & -1 \\ 2 & 3 & -2 \\ -1 & -1 & 1 \end{bmatrix}}$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.