Math 1553 Exam 3, Summer 2026, Ver. A

Name	Key	GT ID		Section	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Nurhan (D, 12:30 PM)   Barone (E, 2:00 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- Simplify your answers.
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 2:00 PM on Tuesday next week, after the make-up exam.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

(a) If A is a 3×3 matrix and $\det(A) = -1$, then $\lambda = 0$ is not an eigenvalue of A.

● True

IMT DLC

○ False

(b) If A and B are invertible $n \times n$ matrices, then $\det(A + B) = \det(A) + \det(B)$.

○ True

$A = B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

● False

(c) If A is an 2×2 matrix and $\det(A) = 2$, then $\det(2A) = 8$.

● True

and.

e.g. $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $\det cA = c^n \det A$

○ False

(d) Suppose A is a 3×3 matrix and

$$A \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \\ 4 \end{pmatrix}. \quad \lambda = 4$$

Then the null space of $A - 4I$ must include at least one nonzero vector.

● True

$\lambda = 4$ eigenspace

○ False

(e) Suppose A is an $n \times n$ matrix and that u and v are linearly independent eigenvectors of A corresponding to the same eigenvalue of A. Then $u + v$ must be an eigenvector of A.

● True

can't be $\vec{0}$

○ False

2. Multiple choice. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Find A^{-1} for $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

$A^{-1} = \frac{1}{4-6}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \frac{1}{2}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$

○ $A^{-1} = \begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$ ○ $A^{-1} = \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ ○ $A^{-1} = \begin{pmatrix} -4 & 3 \\ 2 & -1 \end{pmatrix}$

● $A^{-1} = \frac{1}{2}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$ ○ $A^{-1} = \frac{1}{2}\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ ○ $A^{-1} = \frac{1}{10}\begin{pmatrix} -4 & 2 \\ 3 & -1 \end{pmatrix}$

○ none of these

$A = (v_1 \ v_2) = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$

(b) (2 points) Find the area of the parallelogram with vertices

$$(1,1), \quad \underset{v_1}{(3,2)}, \quad \underset{v_2}{(2,5)}, \quad (4,6).$$

$\det A = 8 - 1 = 7$

Clearly fill in the bubble for the correct answer below.

○ $\frac{7}{2}$ ○ 5 ○ 6 ○ $\frac{9}{2}$ ● 7 ○ 9 ○ none of these

$3R_1$ then R_1

(c) (2 points) Suppose $\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = -2$. Find $\det \begin{pmatrix} -2d+3a & -2e+3b & -2f+3c \\ d & e & f \\ 4g & 4h & 4i \end{pmatrix}$.

Clearly fill in the bubble for the correct answer below $4R_3$

○ -12 ○ 12 ○ -16 ○ 16 ○ -20 ○ 20

● -24 ○ 24 ○ -48 ○ 48 ○ none of these

$-2 * 3 * 4$

$= -24$

(d) (4 points) Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$. Which of the following are eigenvectors of A? Clearly fill in the bubble for all that apply.

● $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ ● $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\lambda = 3$ $\lambda = 0$ X X

3. Multiple choice. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Let T be the linear transformation $T(x, y) = (3x - 2y, x - 4y)$, and let S be a rectangle in $\mathbf{R}^2$ with area 3. Find the area of $T(S)$.

○ 10 ○ 13 ○ -6 ● 30 ○ -30

○ 42 ○ 48 ○ 3/10 ○ none of these

$A = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix}$

$\det A = -12 + 2$

$= -10$

(b) (2 points) Let $A = \begin{pmatrix} -2 & 0 & 1 \\ 4 & 1 & -3 \\ 0 & 0 & 3 \end{pmatrix}$.

Find $\det(A^2)$. Clearly fill in the bubble for your answer below.

○ -6 ○ 6 ○ -12 ○ 12 ○ -16 ○ 16

○ -36 ● 36 ○ -64 ○ 64 ○ none of these

$\det \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix} = -6$

(c) (3 pts) Suppose A is a 2×2 matrix and the vectors $v_1 = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ are eigenvectors of A with eigenvalues $\lambda_1 = 2$ and $\lambda_2 = -1$. Which of the following statements must be true? Clearly fill in the bubble for all that apply.

● A is invertible.

● The image of $\begin{pmatrix} -1 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ 3 \end{pmatrix}$ under the transformation $T(x) = Ax$ is $\begin{pmatrix} -6 \\ 1 \end{pmatrix}$.

● $A = \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix} \begin{pmatrix} 4 & -1 \\ 3 & 2 \end{pmatrix}^{-1}$.

$2\begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$

(d) (3 points) Which of the following matrices have the property that their 3-eigenspace is a plane? Clearly fill in the bubble for all that apply.

● $\begin{pmatrix} 3 & 1 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ ○ $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{pmatrix}$ ● $\begin{pmatrix} 3 & 0 & 1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

4. On this page, you do not need to show your work. Parts (a)-(d) are unrelated.

(a) (2 points) Suppose A and B are 3×3 matrices with $\det(A) = 2$ and $\det(B) = 5$. Find $\det(2A^{-1}B)$. Fill in the bubble for your answer below.

○ 5 ○ $\frac{5}{16}$ ○ 10 ● 20 ○ 80 ○ none of these

$2^3 * \frac{1}{2} * 5 = 20$

(b) (4 points) Let $A = \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix} \begin{pmatrix} 1/3 & 0 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}^{-1}$.

(i) Find $A\begin{pmatrix} 3 \\ 2 \end{pmatrix}$. Fill in the bubble for your answer below.

○ $\begin{pmatrix} -6 \\ -4 \end{pmatrix}$ ● $\begin{pmatrix} 1 \\ 2/3 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ -2/3 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

○ $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ○ none of these

(ii) Find $A^3\begin{pmatrix} -1 \\ 5 \end{pmatrix}$. Fill in the bubble for your answer below.

$(-2)^3\begin{pmatrix} -1 \\ 5 \end{pmatrix} = -8\begin{pmatrix} -1 \\ 5 \end{pmatrix}$

○ $\begin{pmatrix} 2 \\ -10 \end{pmatrix}$ ○ $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ -15 \end{pmatrix}$

○ $\begin{pmatrix} -8 \\ 40 \end{pmatrix}$ ● $\begin{pmatrix} 8 \\ -40 \end{pmatrix}$ ○ $\begin{pmatrix} -27 \\ 135 \end{pmatrix}$ ○ none of these

(c) (2 points) Suppose A is a 5×5 matrix with characteristic polynomial

$$\det(A - \lambda I) = -(\lambda - 4)^2(\lambda - 5)(\lambda - 6)^3.$$

Which **one** of the following statements is true?

○ It is possible for the null space of A to be a line.
● $\dim(\mathrm{Nul}(A - 5I))$ must equal 1.
○ $\det(A - 4I) = 2$.
○ If the 4-eigenspace of A has dimension 2, then A must be diagonalizable.

(d) (2 points) Let A be the 2×2 matrix that reflects every vector $\begin{pmatrix} x \\ y \end{pmatrix}$ in $\mathbf{R}^2$ across the line $y = 3x$. Which **one** of the following vectors is in the 1-eigenspace of A? Clearly fill in the bubble for your answer.

○ $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$ ○ $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ ● $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$

5. (10pt) Free response. Show your work unless otherwise indicated! A correct answer without appropriate work will receive little or no credit.

For this problem, let $A = \begin{pmatrix} 3 & 6 & -12 \\ 0 & 1 & 4 \\ 0 & 0 & 3 \end{pmatrix}$.

(a) Write the eigenvalues of A. You do not need to show your work on this part. $\boxed{1, 3}$

(b) For each eigenvalue of A, find a basis for the corresponding eigenspace. Put your answers in the boxes provided.

$\lambda = 1$

$$A - I = \begin{bmatrix} 2 & 6 & -12 \\ 0 & 0 & 4 \\ 0 & 0 & 2 \end{bmatrix} \sim \begin{array}{c} \quad\ \ x_2 \\ \begin{pmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \end{array} \quad x = x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

basis is $\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$

$\lambda = 3$

$$A - 3I = \begin{bmatrix} 0 & 6 & -12 \\ 0 & -2 & 4 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{array}{c} x_1 \qquad\quad x_3 \\ \begin{pmatrix} 0 & 1 & -2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{array}$$

$$x = x_1 \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

basis is $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$

λ_1-eigenspace basis

$$\left\{ \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} \right\}$$

λ_2-eigenspace basis

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$$

(c) The matrix A is diagonalizable. Write a 3×3 matrix C and a diagonal matrix D so that $A = CDC^{-1}$. You do not need to show your work on this part.

$$C = \begin{pmatrix} -3 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

6. Free response. Show your work unless otherwise indicated! A correct answer without appropriate work will receive little or no credit. Parts (a) and (b) are unrelated.

(a) (4 points) Find $\det\begin{pmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & 1 & -1 \\ 0 & 0 & -2 & 0 \\ 1 & 2 & 0 & 1 \end{pmatrix}$.

Enter your answer here: -18.

$$\det A = -2 \det \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

$$= -2\left(1 \det\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix} + 2\begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\right)$$

$$= -2\left((1+2) + 2(4-1)\right) = -2(3+6) = -18$$

(b) (6 points) Find the *complex* eigenvalues of the matrix $A = \begin{pmatrix} 2 & -4 \\ 2 & 6 \end{pmatrix}$. For the eigenvalue with **negative** imaginary part, find a corresponding eigenvector v. Simplify your eigenvalues as much as possible! Enter your answers below.

$12+8=20$

The eigenvalues are: $4 \pm 2i$ $\qquad v = \begin{pmatrix} -1-i \\ 1 \end{pmatrix}$

$$p(\lambda) = \det(A - \lambda I) = \lambda^2 - 8\lambda + 20 = 0$$

$$\Rightarrow \lambda = \frac{8 \pm \sqrt{64-80}}{2} = 4 \pm \frac{\sqrt{-16}}{2} = 4 \pm 2i$$

$\lambda = 4-2i$

$$A - (4-2i)I = \begin{bmatrix} 2-(4-2i) & -4 \\ 2 & 6-(4-2i) \end{bmatrix} \sim \begin{bmatrix} 1 & \frac{2+2i}{2} \\ 0 & 0 \end{bmatrix}$$

x_2

$$x = x_2 \begin{pmatrix} -1-i \\ 1 \end{pmatrix}$$

7. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work will receive little or no credit. Parts (a) and (b) are unrelated.

(a) (6 points) Let $A = \begin{pmatrix} 0.7 & 0.2 \\ 0.3 & 0.8 \end{pmatrix}$.

i. Find the steady-state vector w for A. Enter it in the space below.

$$A - I = \begin{bmatrix} -.3 & .2 \\ .3 & -.2 \end{bmatrix} \sim \begin{bmatrix} 3 & -2 \\ 0 & 0 \end{bmatrix}$$

x_2

$$\sim \begin{bmatrix} 1 & -2/3 \\ 0 & 0 \end{bmatrix} \quad X = x_2 \begin{bmatrix} 2/3 \\ 1 \end{bmatrix}$$

set $x_2 = 3$ so $X = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \rightarrow w = \begin{pmatrix} 2/5 \\ 3/5 \end{pmatrix}$

$w = \begin{pmatrix} 2/5 \\ 3/5 \end{pmatrix}$

ii. What vector v does $A^n \begin{pmatrix} 100 \\ 200 \end{pmatrix}$ approach as n gets very large? Enter your answer in the space below. Briefly show your work.

$N = 100 + 200 = 300$

So $A^n \begin{pmatrix} 100 \\ 200 \end{pmatrix} \approx N \begin{pmatrix} 2/5 \\ 3/5 \end{pmatrix}$

$v = \begin{pmatrix} 120 \\ 180 \end{pmatrix}$

(b) (4 points) Find a 2×2 matrix A whose 4-eigenspace is the span of $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ and whose (-1)-eigenspace is the span of $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$. Write your answer in the space below.

$$A = CDD^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{3-2} \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 12 & -4 \\ 2 & -1 \end{pmatrix} = \begin{pmatrix} 14 & -5 \\ 30 & -11 \end{pmatrix}$$

$A = \begin{pmatrix} 14 & -5 \\ 30 & -11 \end{pmatrix}$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.