Worksheet 2

1. Consider the following vectors:

$$\vec{u}_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \ \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \ \vec{u}_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \ \vec{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

   Is $\vec{b}$ in the span of $\{\vec{u}_1, \ \vec{u}_2, \ \vec{u}_3\}$? Is $\vec{b}$ a linear combination of vectors $\vec{u}_1, \ \vec{u}_2$, and $\vec{u}_3$? Is the linear system with augmented matrix $A = \left[\vec{u}_1 \ \vec{u}_2 \ \vec{u}_3 \mid \vec{b}\right]$ consistent?

2. Mark each statement as true or false, and justify your answers:

   (a) A vector $\vec{b}$ is a linear combination of the columns of a matrix A if and only if the equation $A\vec{x} = \vec{b}$ has at least one solution.

   (b) The equation $A\vec{x} = \vec{b}$ is consistent if the augmented matrix $\left[A \mid \vec{b}\right]$ has a pivot position in every row.

   (c) The first entry in the product $A\vec{x}$ is a sum of products.

   (d) If the columns of an $m \times n$ matrix A span $\mathbb{R}^m$, then the equation $A\vec{x} = \vec{b}$ is consistent for each $\vec{b}$ in $\mathbb{R}^m$.

   (d) If A is an $m \times n$ matrix and if the equation $A\vec{x} = \vec{b}$ is inconsistent for some $\vec{b}$ in $\mathbb{R}^m$, then A cannot have a pivot position in every row.