Worksheet 3

1. Find the solution of the following system and write it in parametric vector form. Give a geometric description of the solution set.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3 \end{aligned}$$

2. For the following situations, determine (a) whether the equation $A\vec{x} = \vec{0}$ has a nontrivial solution and (b) whether the equation $A\vec{x} = \vec{b}$ has at least one solution for every possible $\vec{b}$ in $\mathbf{R}^m$, and explain:

   (i) A is a 3×3 matrix with 3 pivots. **Solution**: (a) A has a pivot in every column, so there are no nontrivial solutions since there are no free variables. (b) A has a pivot in every row, so there is at least one solution for every $\vec{b}$ in $\mathbf{R}^3$ - in fact, there is exactly one solution for every $\vec{b}$, since there are no free variables.

   (ii) A is a 3×3 matrix with 2 pivots.