Worksheet 4

1. Consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 4 \\ -7 \\ 9 \end{bmatrix}, \ \vec{v}_2 = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \ \vec{v}_3 = \begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix}$$

Note that the sum of $\vec{v}_1$ and twice $\vec{v}_2$ is equal to $\vec{v}_3$. Denote A as the matrix $A = [\vec{v}_1 \ \vec{v}_2 \ \vec{v}_3]$. Answer the following questions, with justification:

   (a) Is the set of vectors $\{\vec{v}_1, \ \vec{v}_2, \ \vec{v}_3\}$ linearly independent?

   (b) Without row reducing, does A have a pivot in every column?

   (c) Does the linear transform $T(\vec{x}) = A\vec{x}$ map $\mathbb{R}^3$ onto $\mathbb{R}^3$? Is this linear transform one-to-one?

   (d) Again, without row reducing, find a non-trivial solution of $A\vec{x} = \vec{0}$.

2. Let the linear transform $T : \ \mathbb{R}^2 \to \mathbb{R}^2$ first reflect points across the horizontal x_1 axis and then reflect them across the line $x_2 = x_1$. Find the standard matrix representation of T.